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The Design of Debt Clearing Markets: Clearinghouse Mechanisms in Pre-Industrial Europe

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Abstract

We examine the evolution of the decentralized clearinghouse mechanisms that were in use throughout Europe from the 13th century to the 18th century; in particular, we explore the clearing of non- or limited-tradable debts like bills of exchange. We construct a theoretical model of these clearinghouse mechanisms and show that the specific decentralized multilateral clearing algorithms known as *rescontre*, *skontrieren* or *virement des parties*, used by merchants in this period, were efficient in specific historical contexts. Our analysis contributes to the understanding of these mechanisms during late medieval and early modern fairs and their robustness during the 17th and 18th centuries.

JEL classification: C78; D02; D47; N23

Keywords: Market design; Matching; History of decentralized clearinghouses; History of central exchange banks

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1 Introduction

Studying pre-industrial institutions is crucial to understanding the long run growth of Europe. During this era, the intensification of regional and long distance trade required the creation of institutional solutions to address the difficulties arising from more complex financial arrangements. Indeed, many of the institutions which form the foundation of the modern economy were first employed in more primitive forms during the late Middle Ages or Early Modern era: this is particularly true for institutions related to trade (Lopez 1976; Greif 2006).

An important step in the evolution of trade was the separation of *quid* and *quo* (Greif 2002, 2006b). That is, during this era, the payment for purchased goods no longer necessarily took place at the same time and location as the delivery of the goods but instead could take place at the next fair or market, possibly at a different location. Alongside this development appeared financial instruments like the *cambium*, *letter obligatory*, or the *bill of exchange*, which were promises of future payment at a specific time and place by the drawer or his representative or banker (Goldschmidt 1891; Schapps 1892; de Roover 1953). Access to such instruments was crucial to a merchant, as such access allowed the merchant to forestall payment until a time and place that he expected to be liquid again (e.g., after the successful trading of goods). Moreover, cashless payment was a good alternative to carrying heavy gold or silver coins along dangerous trading routes; it also limited the dependency of merchants on the currency policies of local dukes. Finally, the use of cashless payment helped to overcome the money shortage of the Late Middle Ages caused by the scarcity of silver and gold bullion (Miskimin 1984; Spufford 1988).

The development of cashless trade necessitated the establishment of clearing institutions, through which liabilities could be settled for merchants and merchant bankers from all over Europe. However, any clearing institution faced a number of significant challenges: First, an agent may lack the liquid assets necessary for the direct payment of debts, even though his net position is positive, a condition referred to as *time mismatch*; this may hinder yet

other merchants' abilities to clear their positions. Moreover, this may hinder trade activity by creating "gridlock," i.e., situations where a merchant is unable to engage in surplus-enhancing trade due to liquidity constraints. Second, there may be *limited enforcement* of contracts, that is, it may be difficult to compel payment of debts by debtors. Indeed, these issues have been identified in the literature on contemporary payment mechanisms as key problems to solve (Kahn and Roberds 2009).

In developed economies, centralized clearing institutions backed up by strong legal institutions and enforcement mechanisms reduce the problems of time mismatch and limited enforcement, creating rather robust clearing mechanisms.¹ Additionally, these problems are alleviated by the buying and selling of debt instruments. However, financial instruments, such as bills of exchange, were in general not tradable due to the limited enforceability of financial contracts across Europe (Usher 1943; Van der Wee 1963; North 1981; Munro 1994).² Hence, settlement could not simply occur through the buying and selling of bills of exchange; instead, a specific institution designed to efficiently clear debt contracts in the presence of this constraint was necessary.

In fact, a self-enforcing clearing mechanism to offset non-tradable credits and debts was developed during this era, known in the sources as *rescontrire*, *rescontre*, *skontrieren*, *vivre compte*, or *virement des parties*. At the end of a market period, all merchants with liabilities met in order to clear those liabilities.³ During this procedure, each merchant revealed his debtors, and each creditor had to confirm his liabilities. The merchants then tried to offset credits with debts through a decentralized searching and matching procedure: in the first step they canceled reciprocal debts and in the second step they used clearing cycles and chains. Once no more chains or cycles could be found, any liabilities still extant had to be paid in cash or new bills had to be drawn.⁴

¹For instance, see Bech et al. (2012) for an overview of the financial architecture in the United States. For a discussion of clearing failures in modern times, see Fleming and Garbade (2005).

²See Börner and Ritschl (2009, 2011) for a discussion.

³The unit of account was either an artificial fair currency or the local currency; exchange rates were agreed to before any clearing of liabilities took place.

⁴A complete description of the mechanism is given in Section 2.2.

By formally modeling the rescontre mechanism, we show that this decentralized clearing procedure has several appealing properties. We first consider the case of balanced positions, where each agent's liabilities are exactly offset by his assets.⁵ In this case, the rescontre procedure was sufficient to obtain an efficient outcome. Moreover, when positions are balanced, it is sufficient to clear only reciprocal debts and cycles; hence, no agent has to take on a new debtor at any point in the procedure.

We then consider the more general case where a given merchant may be a net creditor or net debtor. However, a natural question arises in such situations: If a creditor agreed to cancel a debt with his debtor in exchange for the debt of another, was his original debtor liable in the case that the new debtor was unable to pay? To facilitate clearing, it was essential that the answer to this question was negative, i.e., debt transfers were final.⁶ Thus, creditors would be very careful about canceling such debts and taking on new debtors. Hence, it was vital that creditors were given the right to refuse to exchange one debtor for another, as otherwise they would have strong incentives not to participate in the clearing.

We show that, when an agent can block a chain from clearing, the mechanism is no longer fully efficient in general. However, the voluntary nature of the mechanism implies that it is still the case no agent will be forced into a transaction from which he does not benefit, providing strong incentives for agents to truthfully reveal their positions. Moreover, in the simple case where there are only two types of agents, i.e., where debtors are either "good" or "bad," a mechanism which only clears chains that are approved by all the members of the chain is sufficient to obtain a Pareto efficient outcome. Finally, even when there is a greater degree of heterogeneity among debtors, we show that simply clearing chains is enough to obtain a constrained form of efficiency.⁷ In this way, the rescontre procedure enables the clearing of debts in a heterogeneous environment with limited enforcement options.

⁵This could be achieved, for instance, by requiring each merchant to pay any outstanding debts at the end of the clearing process in cash.

⁶This condition is referred to as *finality of payments* in the payments literature: see Kahn and Roberds (2007).

⁷See Section 3.3 for a formal definition of constrained Pareto efficiency.

These properties facilitated the spread of the *rescontre* procedure throughout Europe. At first, merchants and merchant bankers met at the end of a fair to settle bills of exchange and other liabilities. Later, permanent payment days were established and, during the 16th century, exclusive clearing fairs were initiated in Besancon, Piacenza, and Novi, where bills of exchange were settled from all over Europe.⁸ This clearing mechanism continued to develop at late medieval fairs and was perfected at the financial fairs of the late 16th and early 17th centuries, where it developed into a formal procedure embedded in a legal framework. It was the dominant debt-clearing mechanism in Europe's most important trade centres: Germany, Italy, France, the Low Countries, and Spain. These markets and fairs were not only the central nodes of European finance and trade but also facilitated the acquisition of capital to finance trade with the New World.⁹ These clearing mechanisms waned in importance when the first exchange banks took over the clearing business and general endorsement of bills of exchange during the 17th century (Schneider 1989; Brübach 1994).

For a comparative perspective, we also consider alternatives to the *rescontre* procedure used during the early Modern period: clearing clubs and public exchange banks. During the early Modern period, clearing clubs restricted access to the *rescontre* process in order to reduce heterogeneity among participants. This reduction in heterogeneity improved efficiency for some participants but reduced access for other participants.

On the other hand, public exchange banks could always implement a Pareto efficient allocation with respect to the reported debt positions. However, an agent who cleared his liabilities through a central bank relied on the reserves of the bank. In effect, this agent was relying on the entire pool of merchants who had outstanding obligations to the bank.

We show that a pure exchange bank, i.e., a bank which only clears positions, will fail since only the worst quality debts will be brought to the bank. Merchants with good quality

⁸See the work of Endemann (1885), Neuhaus (1892), Ehrenberg (1896), Vigne (1903), Haristoy (1906), Prausnitz (1928), Carande (1943), Lapeyre (1955), Da Silva (1969), Gascon (1971), Boyer-Xambeu et al. (1986), Denzel (2005, 2008), Marsilio (2008).

⁹See the references cited above regarding the locations of exclusive clearing fairs as well as the work of Pezzolo and Tattaro (2008).

debts will instead endeavor to clear those debts directly. Intuitively, a merchant who brings a high quality debt to the bank will be forced to pool that debt with lower quality debt, thus lowering that merchant's expected payoff from that debt. However, an exchange bank may be able to create a profit stream based on the deposits of its customers; in this case, this profit stream provides a positive incentive for merchants to participate. We show that, when such a profit stream is present, there may exist multiple equilibria, where participation is endogenous and depends on the participation of customers who bring high quality debts to the bank, i.e., debts with low likelihoods of default. However, achieving this outcome relies on the trust of the (high quality) customers in the banking institution. We show that these banking characteristics and institutional properties can be documented in the early successful exchange banks of Amsterdam and Hamburg.¹⁰

Successful central banks required the ability to create a profit stream in order to engender trust by merchants; such strong institutional requirements implied that decentralized clearing mechanisms coexisted with central banks during the early Modern period. In fact, there is strong evidence that the decentralized procedure was still in use at many markets and fairs during the 17th and 18th centuries. In addition, traders who worked in the first stock exchange markets used similar mechanisms to offset payments for stocks or future contracts in cities like Amsterdam and London (Wilson 1941; Dickson 1967; Gelderblom and Jonker 2005). Similarly, after the establishment of the London Clearing Club in 1775, banks in England and on the European continent began settling their liabilities in a similar decentralized way (Seyd 1872). Thus, this decentralized multilateral clearing mechanism was the backbone of financial clearing during more than 500 years of pre-industrial growth in Europe.

While scholars have long recognized that this clearing technology was used for cashless payment all over Europe in order to generate credit and solve problems from money shortages (Neuhaus 1892; Ehrenberg 1896; Boyer-Xambeu at al. 1986; Brübach 1986; North 1996;

¹⁰Our analysis of the central banking mechanism is only concerned with liability issues so as to facilitate comparison with the *rescontre* mechanism. Hence, we abstract away from other important banking functions such as monetary policy: For a discussion of these, see Quinn and Roberds (2006, 2007, 2014).

Denzel 2008; Marsilio 2008), they have not analyzed the efficiency or strategic properties of the mechanism. Instead, legal and economic historians have focused on description, and have either assumed that the mechanism was efficient due to its success, or argued that the mechanism was inefficient as the clearing was decentralized and seemed overly complex. Another strand of literature has focused on the evolution of public exchange banks: see van Dillen (1934), Gillard (2004), Dehing (2012), Denzel (2012), and Roberds and Velde (2014), among others. Besides providing descriptive analysis of these banks, a recent literature has also focused on their monetary policy; see, in particular, Quinn and Roberds (2006, 2007, 2014). However, this literature does not provide a systematic comparative analysis of the decentralized *rescontre* procedure to central banks.¹¹

By contrast, a wide literature on modern payment and clearing mechanisms does analyze the strategic behavior by agents induced by different mechanisms.¹² Moreover, the choice of clearing procedure can have significant effects on trade and welfare (Freeman 1996; Kahn and Roberds 2001). A central question in this literature is the use of *gross* versus *net* clearing procedures. In our context, the lack of a central institution, such as a public bank with sufficient trust and enforcement power, implies that gross clearing procedures were not feasible before the 17th century. This lack of a central institution necessitated the use of a net clearing procedure which does not require a trusted central authority, e.g., the *rescontre* procedure. However, the use of a net clearing procedure, such as *rescontre*, implies that participants are at risk from the default of others. Thus, the allocation of liabilities for unpaid debts is a key question for any such clearing procedure: for the *rescontre* mechanism to be successful, it was necessary that acceptances and rejections of chains and cycles were voluntary and transfers were final.¹³

More generally, this paper follows the tradition established by Avner Greif and others

¹¹A related study, which goes beyond our period of investigation, is the analysis of decentralized and centralized clearing mechanisms in futures markets during the 19th century by Moser (1998).

¹²See, for example, the work of Freixas and Parigi (1998), Furfine and Steam (1998), Kahn and Roberds (1998), and Angelini (1998).

¹³For further discussion of this point, see Section 2.2.

(Greif 1993, 2006; Greif et al. 1994) in that we provide a formal analysis of an important historical institution in order to understand long-run economic outcomes. In particular, we contribute to the understanding of the shaping of historical institutions through the lens of market design, and in so doing help to explain the design of pre-industrial financial institutions in Europe. These financial institutions were a key ingredient of European economic growth, as such growth was mainly based on trade (de Roover 1953; van der Wee 1966; Lopez 1976; Cipolla 1994; Munro 2002; Fratiani and Spinelli 2006). Finally, this paper builds on many of the insights from the market design literature, particularly those works which consider allocation mechanisms without transfers; see, e.g., Pápai (2000), Abdulkadiroğlu and Sönmez (2002), Ehlers et al. (2002), Roth et al. (2004, 2005), and Hatfield (2008).

The remainder of the paper is structured as follows: Section 2 examines the history of the mechanism, from its origination in the late Middle Ages to the development of formal procedures and legislation in the 16th and 17th centuries; we also evaluate the efficiency of the mechanism based on descriptive source analysis. Section 3 studies the *rescontre* mechanism more formally: A game-theoretic model describing the *rescontre* procedure is constructed and equilibrium outcomes are discussed. In Section 4, the theoretically-predicted outcomes are discussed in their historical context. Section 5 compares exchange banks and clearing clubs to the *rescontre* mechanism. Section 6 concludes. All proofs are in Appendix A or Appendix C.

2 Historical Development

2.1 Early Development and the Formation of a Formal Procedure

Evidence of the use of debt clearing mechanisms such as *rescontrire* or *rescontre* procedures can be traced to the late Middle Ages. The earliest documents do not inform us about the detailed structure or procedures of the clearing process, but rather just indicate its existence and probably its common use. The earliest sources are from Champagne fairs at the end

of the 12th century and beginning of the 13th century (Anschütz 1887, pp. 108f.; Haristoy 1906, pp. 284-88; Face 1957). The use of mutual settlement at the central fair locations of Europe can also be documented from the late 14th century and the 15th century onwards: in Germany at Frankfurt, Brunswick, and later Leipzig (Orth 1765, p. 446; Wolf 1967, p. 205f.; Bücher 1915, p. 212f.; Brübach 1994, p. 284, 307), in France at Lyon (Ehrenberg 1886, pp. 74-8; Vinge 1903, pp. 110ff.; Haristoy 1906, pp. 296-301; Bresard 1914; Gascon 1971, pp. 242-8; Boyer-Xambeu 1986, pp. 189-198), in the Netherlands at Antwerp and Bergen op Zoom (Ehrenberg 1896, pp. 10-13; van der Wee 1993, pp. 151-158), and in Spain at the Castilian fairs of Medina del Campo (Rodriguez and Fernandez 1903, pp. 642f.; Carande 1943, pp. 195-234.; Martin 1965, pp. LXXXII-CIII; Vasquez de Prada 1991, pp. 117f). Early evidence from the 15th and 16th centuries for the use of the *rescontre* procedure can also be found in economically important Mediterranean merchant cities such as Barcelona, Genoa, Naples, and Venice (Capmany 1792, p. 167; Lattes 1869, pp. 72; Ajello 1884, pp. 728ff.; Prausnitz 1928, p. 11).

The first regulations describing the clearing procedure in detail are from 16th century Piacenza; these regulations provide further evidence of the common use of this clearing technology in many cities and fair locations during the 15th and 16th centuries. These regulations, from 1597, were taken from the clearing fairs of Besancon and are called the orders of Besancon. The clearing fairs at Besancon were the first exclusive clearing fairs and were founded by the Genoese in 1537 to compete with and replace the clearing fairs of Lyon, which in turn go back to the second half of the 15th century.¹⁴ These orders from Besancon were recorded by Raphael Turri in 1642, who describes the clearing process in detail.¹⁵

These regulations can be seen as a blueprint for the procedures found in many exchange

¹⁴Customary practices of the 16th century Lyon fair have been handed over by Fabiano (1556), Trenchant (1556), de Savonne (1567), and de Rubys (1604). Later authors include Boyer (1627) and Clerac (1656). See Vigne (1903, pp. 118-128), Haristoy (1906, pp. 296-301), Bresard (1914), and Gascon (1971, pp. 242-8). References to the original source material of contemporary writers of the pre-modern time can be found in Appendix B.

¹⁵Peri (1638) and Scaccia (1645) also give descriptions of the clearing process (Neuhaus 1892; Endemann 1885, 174ff.; Ehrenberg 1896, pp. 230-6; Prausnitz 1928; Da Silva 1969, Chapter 2).

laws during the 17th and 18th centuries in Europe. Similar procedures can be found at the fairs in Italy, such as those at Novi, Bozen, and Verona, which replaced Piacenza (Da Silva 1969; Denzel 2005, 2009). In France, Lyon remained the clearing center for bills and other obligations, where the procedures were codified in detail in 1667 based on earlier attempts starting in 1642 (Haristoy 1906, pp. 303ff; Vigne 1903, pp. 118ff.). At the same time, Frankfurt, the most important fair location and finance center in Germany, produced a more detailed exchange law, inspired by the codification in Lyon. Whereas Frankfurt already had a long tradition of using this clearing technique going back to the late 14th century, clearing regulations could only be found from 1578 and 1611, along with a more detailed version from 1666 (Brübach 1994, pp. 169-76). Other important clearing locations with regulations concerning the mechanism were Brunswick (with regulations from 1686 and 1715) and Leipzig (with regulations from 1682 and 1711).¹⁶ Further evidence is available from merchant cities such as Augsburg, Breslau, Nuremberg and others (Brübach 1994, p. 318; Neuhaus 1892, p. 11).¹⁷ Although we only have fragmented evidence of the use of the mechanism, these observations indicate that the *rescontre* mechanism was used in most of the important trade hubs in Europe.

2.2 Regulations and Customary Practices

Clearing took place about four times a year. This goes back to the tradition that fairs were held quarterly and, at the end of each fair, a financial clearing was organized.¹⁸ This pattern can be documented for Lyon, the Brabant fairs at Antwerp and Bergen, the Castilian fairs and all the Italian and most of the German fairs (Ehrenberg 1892; Da Silva 1969; van der Wee 1963; Boyer-Xambeu et al. 1986). In cities in which fairs became permanent markets or financial clearing was separated from goods trading, the frequency was increased to monthly

¹⁶Very short regulations also exist for Leipzig for 1621 and 1676: see Hasse (1886, pp. 277f.), Neuhaus (1892, p. 11), and Brübach (1994, pp. 310ff).

¹⁷Beside the exchange laws themselves, contemporary writers such as Raumburger (1723) and Riccius (1783) provide more evidence on the use of the procedure in these places.

¹⁸For the typical fair's customs, see Verlinden (1971).

or even weekly meetings during the 17th and 18th centuries. This can be documented in the regulations of Leipzig in 1688 and Frankfurt in 1776, or in Augsburg where only a financial exchange was implemented (Neuhaus 1892, p. 11; Prausnitz 1928, p. 22; Brübach 1994, p. 175, 318).¹⁹ These clearing meetings lasted several days. In Besancon, for example, the meeting always took eight days, with a fixed ordering of events.

For most fairs, the regulations did not specify any restrictions on participation. The clearing was open to everybody; however, de Turri reports that in Besancon participants were merchant bankers and only they had the right to participate (Endemann 1889, p. 174; Prausnitz 1928, p. 17).²⁰ Scaccia and Peri write that whoever wanted to participate had to pay a deposit (Endemann 1889, p. 174; Neuhaus 1892, p. 13; Ehrenberg 1896, p. 231). Sometimes merchants attending a fair were forced to take part in the clearing procedure. This was stipulated in the regulations of Bozen and Leipzig (Neuhaus 1892, pp. 31, 39).²¹ These rules excluded cash payments or agreements about later payments *ex ante* by merchants who had not taken part in the mutual clearing procedure. Under the rules of Besancon, merchants who wanted to be paid in cash had to announce this before the clearing process began (Ehrenberg 1892, p. 235; Prausnitz 1928; Neuhaus 1892). However de Turri, Scaccia, and Azorius, another writer from 1625, report that merchants had preferences for mutual clearing and were worried about receiving their payments in cash (Neuhaus 1892, pp. 19, 21; Endemann 1889, p. 179).

The clearing procedure worked as follows: merchant bankers met with their ledgers (*scartefacci*). In early Besancon there were at least 50-60 bankers; in Piacenza there were about 110 (Ehrenberg 1892, p. 229). Merchants who wanted to participate had to be present or have an official representative. This can be found in the statutes of Besancon and most of the later regulations during the 17th and 18th centuries as well (Endemann 1885, p. 175;

¹⁹Permanent marketplaces document weekly meetings earlier on—for example, sources from Naples indicate weekly meetings as early as the 16th century: see Ajello (1884, pp. 728ff).

²⁰Furthermore, Da Silva (1969) documents a returning core of merchant bankers going to the Italian clearing fairs and Velinov (2012) a steady group of cashiers on the market in Antwerp.

²¹See also the report by Raumburger (1723).

Brübach 1994, pp. 306f.; Neuhaus 1892, p. 30; Haristoy 1906, p. 304f.). Once the set of participants had been determined, all creditors revealed their debtors.²² The debtors had to confirm or reject the debts.²³ In the next step, each banker had to create a balance sheet with all the accepted claims, which had to be submitted in Besancon where officials would check the general balance.²⁴ This practice cannot be documented for other, later, fairs. Only accepted bills were valid for the later mutual netting procedure. Rejected bills were discussed in front of the fair court and settled later;²⁵ this regulation was also in use in all other fairs (Endemann 1885, p. 175ff.; Prausnitz 1928, p. 17; Brübach 1994, p. 309). De Turri writes that not all bankers were honest: hence, some bankers were willing to go in front of the court in order to settle some bills later (Endemann 1885, p. 177). However, there was an opportunity cost for merchants to dispute bills, since any disputed bill could not be used in the clearing process.

In the next step the exchange rates for the bills were fixed in an official accounting currency. This was done by a group of bankers from different regions who had participated in the clearing process. At the Genoese fairs, the bankers who set the exchange rates came only from different regions in Italy (Endemann 1885, p. 179f.; Ehrenberg 1896, p. 232), while in Lyon, they came from France, Germany and Italy (Gascon 1971, pp. 244f.).²⁶ The regulations of Besancon state that exchange rates should be reasonable. However, based on these rates, other market prices were negotiated ex post (Ehrenberg 1896, p. 234f.). Having

²²It is not clear if this process was public, i.e., the merchants met at a public forum where creditors called out debts and debtors accepted or rejected, or private, i.e., it happened thorough private bilateral conversation: see Ehrenberg (1896, p. 234), Martens (1794, p. 16), Neuhaus (1892, p. 8), Prausnitz (1928, p. 17), Endemann (1886, p. 176), and Brübach (1994, p. 175).

²³See the reports by Scaccia and de Turri (Endemann 1886, p. 176). The same practice was in use in Lyon earlier: see Gascon (1971, pp. 243f.). Later, this practice was used at the Frankfurt and Leipzig fairs: see Haristoy (1906, p. 304) and Brübach (1994, p. 310). Note that revelation process was not trivial: Due to the nature of the bills of exchange, debtors typically had to wait for payment instructions from their partners sitting in distant cities in Europe and, similarly, creditors had to wait for orders to collect liabilities from their partners sitting in distant cities in Europe.

²⁴This was reported by de Turri, Peri, and Scaccia: see Endemann (1886, p. 177) and Ehrenberg (1896, p. 234).

²⁵The rejection of such bills can be documented for the fairs of Lyon from at least the second half of the 15th century: see Haristoy (1906, pp. 297f.).

²⁶De Rubys reports in 1604 that this procedure was already use at the Lyon fairs during the 16th century: see Vigne (1903, p. 121).

participants from different regions set exchange rates collectively was also typical for later regulations. In Frankfurt, where the fixing of the exchange rate can be documented from 1585 onwards, in 1625 legislation was passed after some bankers at previous fair meetings tried to manipulate exchange rates in their favor; it was stated that the exchange rates were to be decided by local exchange brokers and merchant bankers from different regions (Ehrenberg 1896, pp. 246f.; Brübach 1994, pp. 287, 311ff.).

In the final stage, merchants started the actual clearing process, sometimes with the help of brokers.²⁷ This process was organized as a decentralized searching and matching procedure. The codified legal rules in various cities are rather cryptic in describing the process.²⁸ However, contemporary witnesses paint a very lively picture of merchant bankers who were in search of finding clearing partners during the clearing time.²⁹

The decentralized clearing algorithm worked the following way: First, merchant bankers offset their positions in a reciprocal way. So if a merchant banker i owes j and j at the same time owes i , the common amount could be directly cancelled.³⁰ In the next step merchants tried to find clearing cycles and clearing chains. In a clearing cycle, for example, i owes j , j owes k , and k owes i a certain amount of money. The common amount then could also be cancelled.³¹ By contrast, in a clearing chain, where i owes j and j owes k , only j could clear his debts and the amount that could be canceled out had to be transferred into a new debt relationship between i and k . These clearing mechanisms could easily include seven or eight parties (Koenigk 1727, p. 66; Orth 1765, p. 481; E.B.A. 1773, p. 109f.). This clearing process was repeated again and again. For example, k could use the transferred debt in

²⁷The help of brokers can be documented for the German fairs in Frankfurt, Brunswick and Leipzig: see Brübach (1994, p. 316f).

²⁸For example, see the regulations of Frankfurt in 1611, Brunswick in 1686 and 1715, Leipzig in 1688, and Breslau in 1742: see Prausnitz (1928, p. 23) and Brübach (1994, p. 170).

²⁹Evidence can be found in de Turri and Scaccia for the earlier period. For the late 17th and 18th centuries, see Raumburger, Marberger, Riccius, and Zipffel (1701): see Endemann (1886, p. 178), Prausnitz (1928, pp. 17, 23), and Neuhaus (1892, p. 9).

³⁰Scaccia and de Turri report this for the Genoese fairs: see Neuhaus (1892, p. 8), Prausnitz (1928, p. 17), and Endemann (1882, p. 177).

³¹For the Genoese fairs Peri gives an example: see Ehrenberg (1896, pp. 235f). For later fairs see, for example, descriptions by Koenigk and Riccius: see Prausnitz (1928 p. 23) and Neuhaus (1892, p. 35).

the chain and bring it into another clearing chain or cycle. This sequential clearing can be documented in the ledger by Masse in 1611 (Brübach 1994, pp. 305f.). The clearing process lasted until the clearing time came to an end or no merchant could find another willing clearing partner. Unfortunately, we cannot quantify the frequency and proportion of fairs that used only cycles to mutually clear debts versus fairs that also allowed the use of chains. However, the historical record indicates that cycles were used more frequently early on and chains started playing a more important role during the 17th and 18th centuries.

In the exchange regulations, differentiated rules arose about the constraints on participation in the clearing mechanism. In general, the rules gave the merchants the freedom to choose the clearing cycle or chains in which they wanted to participate.³² Moreover, there exists a rule that everybody needs to agree to the specific clearing group in most of the exchange laws from the 17th century.³³ All these sources imply preferences of merchants over participation in different clearing cycles or chains. A writer of a merchant handbook in 1773 reports on the rationale behind this. The author writes “you need to be careful to not replace a good debtor with a bad debtor” (E.B.A. 1773, p. 112). Another regulation that indicates different preferences over the order of clearing procedures is legislation from Frankfurt in 1666 where bankers can first offset their liabilities before those of the customers that they represent (Brübach 1994, p. 176).

After the mutual netting was finished, merchants had to check if they still had outstanding credits or debts. All mutual settlements had to have been written in their *scartefacci* and having all participants do this made the clearing official.³⁴ In general, all transfers made by the mechanism were final. In particular, if the debt from i to j was transferred to a

³²de Turri mentions this as an early practice.

³³See, for example, the exchange rules from Frankfurt in 1666, the Prussian exchange orders in 1684, the Leipziger exchange orders in 1688, the orders of Bozen in 1634, the orders of Brunswick in 1715, the orders of Augsburg in 1716, or the descriptions by Raumburger, Marberger, and Riccius: see Neuhaus (1892, p. 29) and Prausnitz (1928, p. 22).

³⁴de Turri mentions this regulation for Besancon. This regulation is mentioned for later fairs by Koenigk, Raumberger, and Marberger: see Prausnitz (1928, p. 17f.). This regulation can be found in the statutes from Lyon in 1667, Leipzig in 1688 and 1715, Breslau, Augsburg, and Bozen: see Neuhaus (1892, p. 12, 31, 38), Prausnitz (1928, p. 17f., 23), and Brübach (1994, p. 314f.).

new creditor k (in order to cancel some previous debt from j to k), j was no longer liable in any way to k .³⁵ Only in Besancon was it the customary practice that everyone within a clearing chain or cycle was collectively liable.³⁶ Legislation ensured that the payment of debts through the clearing mechanism had the same legal status as paying cash.³⁷ Even when it turned out ex post that one merchant was bankrupt, priority was given to the debts that were settled by the mechanism.³⁸ In practice, this meant that the payment could not be reversed. Thus the mechanism was protected from an ex post unravelling of settled debts.

In the final step the remaining debts would be settled. Controversial bills which had been rejected by the debtors were judged by the local court (Ehrenberg 1896, p. 236). The payment of outstanding debts could be in cash or by drawing new bills with promises on future payments within a fixed period.³⁹ In Augsburg in 1778 it was forbidden to draw new bills on one's own name; only cash was allowed (Neuhaus 1892, p. 32). Strict regulations were applied in case a debtor did not offset the outstanding debts by the end of the clearing days: Scaccia and de Turri wrote that participants who did not pay were excluded and declared bankrupt (Endemann 1886, p. 178; Neuhaus 1892, p. 9; Prausnitz 1928, p. 19).⁴⁰ Because of this, Scaccia concluded that merchant bankers with positive positions could easily find new debtors for their money (Endemann 1886, p. 175). By the end of the Besancon fair a general balance was produced by the officials to control the clearing. However such a general balance was not in use at other clearing locations (Endemann 1886 p. 177; Biener 1859 p. 50).

³⁵See, for example, the clauses from Frankfurt in 1666, Leipzig in 1688, Breslau, Brunswick in 1686 and 1715, or reports by Riccius, Marberger, and Koenigk: see Prausnitz (1928, p. 24).

³⁶Contemporary witnesses Fabiani and Merenda inform us of this fact: see Ehrenberg (1896, pp. 235f.), Neuhaus (1892 p. 40), and Brübach (1994, p. 318).

³⁷For example, see the exchange orders of Frankfurt in 1611 and 1666, Lyon in 1667, Bozen in 1634, Brunswick in 1686, Leipzig in 1688, or the contemporary witnesses Franck and Phoosen (Neuhaus 1892, p. 40; Brübach 1994, p. 171, 374).

³⁸See the Besancon regulations, the Statuti di Genoa of 1589, Lyon in 1667, Leipzig in 1688, and the Frankfurt legislation of 1611 and 1666: see Neuhaus (1892 p. 8, 40) and Brübach (1994, p. 171, 309).

³⁹See the statutes of Besancon and Frankfurt in 1666, and the contemporary witnesses de Turri, Raumburger and Franck (Endemann 1886, p. 178; Neuhaus 1892, p. 8; Biener 1859, p. 47; Prausnitz 1928, p. 18, 30; Brübach, p. 175f).

⁴⁰This practice was also already known in Lyon: see Vigne (1904, p. 126) and Haristoy (1906, p. 300). Additionally, in the regulations from 1667 strict payment periods were fixed: see Haristoy (1906, pp. 305f).

2.3 Efficiency of the Mechanism: Evidence from Merchant Books and Contemporary Witnesses

Having outlined the historical evolution of these clearing mechanisms, it remains to be seen whether these mechanisms were efficient in the sense that a high clearing rate could be achieved without paying cash or writing new bills. The existence of the *rescontre* mechanism across Europe, and the preference by merchants for the mechanism, along with the lack of any other institutional solution, indicates that the *rescontre* mechanism must have worked relatively well. Moreover, the large numbers of bills cleared this way gives further evidence (Ehrenberg 1896) that these clearing fairs were successful. Use of these decentralized clearing mechanisms was discontinued only due to economic crises, the economic or political abuse of the institution by manipulating exchange rates or delaying payment, or bankruptcy of major merchant bankers.⁴¹

Another way to evaluate the *rescontre* procedure is to analyze one of the few *scartefacci* which was handed down. The ledger of the 1632 autumn fair in Frankfurt of the merchant banker Johan Bodeck gives detailed evidence of the clearing process at work (Brübach 1994, pp. 331f.). Bodeck was involved with 77 people during the clearing process, which included reciprocal clearing, cycles, and chains. The merchants and bankers involved were from all over northern Europe: Clearing partners came from Aachen, Amsterdam, Augsburg, Breslau, Dresden, Frankfurt, Hamburg, Leipzig, Strasbourg, Ulm and from other unknown locations. During the clearing, he transacted a total amount of 135,000 florins, but only 4,000 florins were paid in cash. The rest were cleared via the *rescontre* procedure. Thus, more than 97% could be cleared by mutual settlement. Taking this single booklet as a sample, the clearing procedure was very successful.

We may also evaluate the decentralized clearing procedures by analyzing the reports of contemporary witnesses. While an early witness, the merchant Davanzati, in 1560 com-

⁴¹See Vasquez de Prada (1991, pp. 121-125) for the Castilian fairs and Denzel (1994, pp. 311ff.) for the Lyon and Italian fairs.

mented on Besancon with some ridicule (Ehrenberg 1896, p. 229), writers of the 17th and 18th centuries, such as Sotus, Molina, Azorius, Scaccia, and de Turri admired the clearing process. Scaccia and de Turri write that nothing was rarer than money at these clearing fairs, as merchant bankers only brought money for their expenses (Endemann 1886, pp. 178f.; Neuhaus 1892, p. 10; Haristoy 1906, p. 300).⁴² Only Koenigk, Raumburger, and Marberger (Prausnitz 1928, p. 25) mentioned that with the emergence of exchange banks, they were surprised that not more places had a centralized exchange bank since they were better than the decentralized clearing procedure. Based on these reports, the mechanism must have been rather successful.

However, if the mechanism was so successful, why did merchants have preferences over which chains and cycles cleared? Moreover, why did the statutes mention the freedom of merchants over the clearing order and the clearing partnerships in which they could be involved? Why did some sets of regulations require that merchants pay off their debts after the clearing process had finished? Finally, why was the mechanism more successful in many instances than alternative clearing institutions such as clearing clubs and clearing banks?

We now turn to a theoretical model of the mechanism to answer these questions.

3 Model

In this section, we describe two different *rescontre* mechanisms which were used to clear debts. The first mechanism, which we call the *rescontre cycles mechanism*, only allows cycles of debts to be cleared, while the second mechanism, the *rescontre chains mechanism*, also allows chains (i.e., where i owes j owes k) to be cleared. While any allocation attainable by clearing cycles is also attainable by clearing chains, we consider the two mechanisms separately as an agent's incentives for agreeing to clear a cycle can be different than those for agreeing to clear a chain. Indeed, the differences in these mechanisms can be used to

⁴²De Rubys and Clairac make a similar point with respect to the Lyon fairs: see Haristoy (1906, p. 300) and Gascon (1971, p. 248).

help understand their use at different times and places; we further discuss the advantages and disadvantages of each mechanism in its historical context in Section 4.

3.1 Framework

There is a finite set of *agents* I . Each agent $i \in I$ is characterized by his *default probability* $\alpha_i \in [0, 1]$. We denote by d_j^i the *debt* that i owes to j , i.e., if $d_j^i > 0$, then i is a debtor to j for the amount d_j^i . An *allocation* is a matrix of debts $\mathbf{D} \equiv (d_j^i)_{i,j \in I}$.⁴³

The *net position* of agent i is given by

$$\nu^i(\mathbf{D}) \equiv \sum_{j \in I} (d_i^j - d_j^i).$$

We will only consider mechanisms which leave the net position of agents unchanged; hence, it is sufficient to define utility for agent i in terms of the debts owed to i . We thus define the *utility* of agent i for an allocation \mathbf{D} as

$$u^i(\mathbf{D}) = - \sum_{j \in I} \alpha_j d_i^j, \quad (1)$$

i.e., the expected loss to agent i from his debtors defaulting.⁴⁴ We model the utility of i as the expected loss from the debtors of i defaulting as, when an agent himself is not bankrupt, that agent will pay all of his outstanding debts and so only the return on debts owed to him affects his utility. If that agent is bankrupt, then he will be indifferent about his debt relationships as he expects any monies owed to him to be seized by his creditors.

An allocation \mathbf{D} is *Pareto efficient* if there does not exist a another allocation $\bar{\mathbf{D}}$ such that the net position of each agent is the same, each agent has weakly higher utility, and some agent has strictly higher utility.

⁴³We require that $d_i^i = 0$ for each $i \in I$, that is, no agent owes himself money.

⁴⁴This expression is equivalent to an expression for utility that is linear in net proceeds when considering a mechanism that leaves net positions unchanged: see Appendix C.2 for a demonstration.

Our restriction that a mechanism leave the net position of each agent unchanged is motivated by the fact that financial instruments were generally not tradable during this period (as described in the Introduction). We also do not consider mechanisms that allow debts to be paid off immediately at a discount, as such arrangements would likely be impractical: Merchants attending these fairs generally did not bring any substantial amounts of coin (Endemann 1886, pp. 178f.; Neuhaus 1892, p. 10; Haristoy 1906, p. 300); this was due to both the relative scarcity of currency (as discussed in Section 2.3) as well as the dangers from carrying money while traveling. Our definition of Pareto efficiency is then motivated by the fact that we consider only mechanisms that leave the net position of each agent unchanged.

We model the *rescontre* mechanism as a decentralized process in which agents, upon meeting, attempt to renegotiate their mutual debt positions in a mutually advantageous way. Throughout, we assume that agents are fully informed regarding the initial allocation and all actions are perfectly observed.⁴⁵

3.2 The Rescontre Cycles Mechanism

We first consider the case of the *rescontre cycles mechanism*. Informally, a cycle is a list of agents such that each agent is a debtor to the next agent on the list and the last agent is a debtor to the first agent on the list. If such a cycle is found, then a new allocation could be generated by clearing the mutual debt; such an allocation would be a Pareto improvement. The *rescontre cycles mechanism* models this process by allowing agents to find cycles and then clear them by mutual agreement.

Formally, given an allocation \mathbf{D} , a *cycle* c is an ordered list of agents $(i_1, i_2, \dots, i_{N(c)})$ such that:

1. Each agent is distinct, i.e., $i_n \neq i_m$ if $n \neq m$ for all $n, m \in \{1, \dots, N(c)\}$.
2. Each agent is a debtor to the next agent, i.e. $d_{i_{n+1}}^{i_n} > 0$ for each $n \in \{1, \dots, N(c) - 1\}$.

⁴⁵This assumption is for simplicity: as we remark below, agents' optimal strategies when possessing less information will still lead to the outcomes identified in Theorems 1, 3 and 4.

3. The last agent is a debtor to the first agent, i.e., $d_{i_1}^{i_{N(c)}} > 0$.

In other words, a cycle is an ordered list of agents such that each agent is a debtor to the agent after him on the list (with $i_{N(c)}$ being a debtor to i_1). We let

$$d(c) \equiv \min \left\{ \min_{n \in \{1, \dots, N(c)-1\}} \{d_{i_{n+1}}^{i_n}\}, d_{i_1}^{i_{N(c)}} \right\},$$

that is, $d(c)$ is the greatest quantity such that each member of the cycle c owes at least that much to the next member of the cycle. For a given allocation \mathbf{D} , the set of cycles is denoted $\mathcal{Y}(\mathbf{D})$.⁴⁶ For simplicity, we consider two cycles to be the same if they differ only by a rotation; e.g., we consider the cycles (i, j, k) , (j, k, i) , and (k, i, j) to all be the same cycle. Note that the set $\mathcal{Y}(\mathbf{D})$ is finite since the set of agents is finite (and each agent can appear only once in any given cycle).

We model the rescontre cycles mechanism as a finite sequential game taking place in discrete time with a state variable $(\mathbf{D}, \mathcal{C})$, where \mathbf{D} is an allocation and $\mathcal{C} \subseteq \mathcal{Y}(\mathbf{D})$. For an initial allocation \mathbf{D} , the initial state of the game is given by $(\mathbf{D}, \mathcal{Y}(\mathbf{D}))$. We also fix an arbitrary ordering over the agents \triangleright .

At each state $(\mathbf{D}, \mathcal{C})$, a cycle $c \in \mathcal{C}$ is proposed.⁴⁷ Then, in order of the ordering \triangleright , each agent involved in the cycle decides whether to accept or reject clearing the cycle. If any agent chooses to reject, the allocation remains unchanged and the game proceeds to the next time period with the new state $(\mathbf{D}, \mathcal{C} \setminus \{c\})$. If every agent chooses to accept, then we *clear* the cycle, obtaining a new allocation $\bar{\mathbf{D}} = (\bar{d}_j^i)_{i,j \in I}$, where

$$\bar{d}_j^i = \begin{cases} d_j^i - d(c) & \text{if } i = i_n \text{ and } j = i_{n+1} \text{ for some } n \in \{1, \dots, N(c) - 1\} \\ d_j^i - d(c) & \text{if } i = i_{N(c)} \text{ and } j = i_1 \\ d_j^i & \text{otherwise,} \end{cases}$$

⁴⁶We denote the set of cycles as $\mathcal{Y}(\mathbf{D})$, instead of the more natural choice $\mathcal{C}(\mathbf{D})$, to distinguish it from the set of chains, introduced in Section 3.3 and denoted by $\mathcal{H}(\mathbf{D})$.

⁴⁷For technical simplicity, we assume that agents know which chain will be proposed for each state $(\mathbf{D}, \mathcal{C})$.

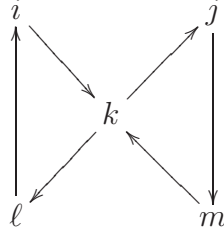


Figure 1: A diagram of a possible allocation \mathbf{D} . An arrow from i to j represents $d_j^i = 100$. There are two cycles: (i, k, ℓ) and (k, j, m) .

and the game proceeds to the next time period with the new state $(\bar{\mathbf{D}}, \mathcal{Y}(\bar{\mathbf{D}}))$; note that the net position of each agent is unchanged. Also note that cycles which were previously rejected may now be reconsidered, since the new state $(\bar{\mathbf{D}}, \mathcal{Y}(\bar{\mathbf{D}}))$ includes all of the feasible cycles at $\bar{\mathbf{D}}$. If the state is of the form $(\bar{\mathbf{D}}, \emptyset)$, then the game ends and final utilities are realized at the allocation $\bar{\mathbf{D}}$. Note that it is clear the game is finite: Clearing a cycle always strictly increases the number of elements equal to 0 in the allocation matrix. Hence, at each step, either the set of still-available cycles is reduced (if some agent rejects the proposed cycle), or a new allocation is formed with strictly more zeroes in the allocation matrix. Since the number of elements of the allocation matrix (i.e., $|I|^2$) is finite, the process must complete in a finite number of steps.⁴⁸

Example 1. As an example of the rescontre cycles mechanism, let $I = \{i, j, k, \ell, m\}$ and consider the allocation depicted in Figure 1. There are two cycles— (i, k, ℓ) and (k, j, m) . Hence, the initial state is given by $(\mathbf{D}, \{(i, k, \ell), (k, j, m)\})$. Suppose that the (i, k, ℓ) cycle is chosen initially; then each of i , k , and ℓ has the chance to approve or reject clearing the cycle. If i , k , or ℓ reject clearing the cycle, then the game proceeds to the state $(\mathbf{D}, \{(k, j, m)\})$. If all three accept, then the new state is given by $(\bar{\mathbf{D}}, \{(k, j, m)\})$, where $\bar{d}_j^k = \bar{d}_m^j = \bar{d}_k^m = 100$, with all other entries of $\bar{\mathbf{D}}$ being 0; note that all three of i , k , and ℓ are strictly better off at $\bar{\mathbf{D}}$ than at \mathbf{D} . At the state $(\bar{\mathbf{D}}, \{(k, j, m)\})$, the only remaining cycle (k, j, m) is proposed: if any of k , j , or m reject the clearing the cycle, then the game ends and the final allocation

⁴⁸If all entries in the allocation matrix D are 0, then $\mathcal{Y}(\bar{\mathbf{D}}) = \emptyset$ and the process ends.

is $\bar{\mathbf{D}}$. If, on the other hand, all three agents accept clearing the cycle, then the game ends with a final allocation of $(0)_{i,j \in I}$.

Note that it is a subgame perfect Nash equilibrium in this example for each agent to always accept any proposed cycle. If every agent does so, then the final allocation is $(0)_{i,j \in I}$; this is the best possible outcome for each agent. Moreover, in any subgame perfect Nash equilibrium, we reach the final allocation $(0)_{i,j \in I}$; if, at any point, the state (\mathbf{D}, C) is such that $|C| = 1$, then each agent in the remaining cycle will choose to accept clearing that cycle, as by so doing, he makes himself better off directly and ensures that the clearing process continues,⁴⁹ which may provide opportunities to increase his utility in the future.⁵⁰

We first consider the cycles mechanism assuming *balanced* positions, that is, where $\nu^i(\mathbf{D}) = 0$ for each $i \in I$; intuitively, when all agents' positions are balanced, the sum of each agent's debts is equal to the sum of the debts owed to that agent. This can be taken as a benchmark case. However, this assumption reflects the reality at many fairs: each merchant going to a particular fair may have planned to have at the end of the trading period a balanced position. More commonly, as documented in Section 2, for many fairs each merchant was forced by fair regulations to balance his position at the end of the fair. For balanced allocations, Example 1 generalizes: in any subgame perfect Nash equilibrium, all of the debts can be cleared by the cycle removal mechanism.

Theorem 1. *For any initial balanced allocation, it is a subgame perfect Nash equilibrium of the rescontre cycles mechanism for every agent to accept the proposed cycle at every step. Moreover, for any balanced allocation, in any subgame perfect Nash equilibrium, the final state is given by $((0)_{i \in I, j \in J}, \emptyset)$.*

To understand the proof of Theorem 1, note that clearing a cycle improves the welfare of each participant directly. Hence, at any step where rejecting a cycle would cause the

⁴⁹Recall that, after a cycle is cleared, previously rejected cycles may be proposed again.

⁵⁰Note that no agent can be made worse off by the clearing process continuing, as he always has the ability to keep his current allocation by not agreeing to clear any future cycles.

mechanism to stop, each member of that cycle is strictly better off (in a myopic sense) by accepting the cycle; moreover, each member of that cycle will be better off when the mechanism finally does end, as clearing a cycle always weakly improves every agent's utility. This shows that the mechanism can not end in equilibrium until the allocation \mathbf{D} is such that $\mathcal{Y}(\mathbf{D}) = \emptyset$, i.e., no more cycles remain to be cleared. However, the only balanced allocation for which no cycles exist is $(0)_{i,j \in I}$; hence, that allocation is the final state of any subgame perfect Nash equilibrium of the rescontre mechanism.⁵¹

Theorem 1 shows that, when positions are balanced, the rescontre cycles mechanism works very well—it obtains the unique Pareto efficient allocation (for which the net position of each agent is unchanged). Moreover, the strategies employed by the agents are very simple—they simply agree to clear any proposed cycle. Furthermore, at each step, agreeing to the cycle immediately improves the utility of each agent in the cycle, even if the mechanism were to end after that step. Hence, regardless of whether agents are fully strategic or myopic optimizers, the mechanism will obtain the Pareto efficient outcome. Similarly, if agents are fully rational but not fully informed regarding the allocation (but do know that the allocation is balanced), it is still optimal for each agent to agree to clear any proposed cycle.

Moreover, the use of the rescontre cycles mechanism does not provide agents with any additional incentives to lie about their debts.⁵² Every transaction must be approved by each agent who is involved, implying that each agent is incentivized to fully disclose his liabilities and assets to the mechanism. This ensures that agents will, in general, truthfully reveal their positions, which is key for the smooth functioning of the mechanism.

However, when positions are imbalanced, it is no longer the case that clearing cycles is sufficient to obtain a Pareto efficient outcome. Consider the simple case where $I = \{i, j, k\}$ and the allocation \mathbf{D} is given in Figure 2a, and suppose that $\alpha_i < \alpha_j$. In that case, no cycle

⁵¹There do exist subgame perfect Nash equilibria where a cycle is rejected; however, as discussed above, once the mechanism reaches a state $(\mathbf{D}, \mathcal{C})$ where $|\mathcal{C}| = 1$, each agent will accept the final cycle in a subgame perfect Nash equilibrium.

⁵²Of course, if a merchant believes that if he disowns a debt he will not have to pay it all, that merchant will have a strong incentive to disown that debt. As discussed in Section 2.2, disputed bills were brought before a court to determine their validity and then had to be settled separately.



(a) Example of an allocation with a chain, but no cycle. An arrow from i to j represents $d_j^i = 100$.

(b) The result of clearing the chain in the allocation of Figure 2a. This allocation is Pareto superior if $\alpha_i < \alpha_j$.

Figure 2: An example economy where the rescontre cycles mechanism is Pareto inefficient.

exists— $\mathcal{Y}(\mathbf{D}) = \emptyset$ —and so the rescontre cycles mechanism has no effect. And yet the initial allocation is not Pareto efficient: both j and k would be strictly better off if the debt of i to j were ‘passed through’ to k , resulting in the allocation in Figure 2b.

Hence, clearing cycles is sufficient to obtain good outcomes when all merchants have balanced positions, but is not sufficient to obtain good outcomes when positions are unbalanced, as exemplified in Figure 2. As we demonstrate in the next section, good outcomes may be obtained in such economies by allowing agents to clear chains in addition to cycles.

3.3 The Rescontre Chains Mechanism

We now consider the case where traders are not required to have balanced positions; for many fairs, merchants were loathe to bring hard currency and might not necessarily wish to exactly balance their credits and debits. In such settings, the rescontre cycles mechanism is insufficient to achieve efficiency, as exemplified in Figure 2. In these cases, rescontre mechanisms which allow for the clearing of chains can improve the outcome.

Formally, given an allocation \mathbf{D} , a *chain* c is a triplet of agents (i, j, k) such that $d_j^i > 0$ and $d_k^j > 0$, i.e., a chain is a triple of agents such that the first agent is a debtor to the second, and the second agent is, in turn, a debtor to the third. We let $e(c) \equiv \min\{d_j^i, d_k^j\}$, i.e., the maximal amount of debt which can be cleared by the chain c . For a given allocation \mathbf{D} , the set of chains is denoted $\mathcal{H}(\mathbf{D})$. Note that the set $\mathcal{H}(\mathbf{D})$ is finite since the set of agents is finite.

We model the *rescontre chains mechanism* as a finite sequential game taking place in discrete time with a state variable $(\mathbf{D}, \mathcal{C})$ such that $\mathcal{C} \subseteq \mathcal{H}(\mathbf{D})$. For a set of agents I and an initial allocation \mathbf{D} , the initial state of the game is given by $(\mathbf{D}, \mathcal{H}(\mathbf{D}))$.

At each state $(\mathbf{D}, \mathcal{C})$, a chain $c \in \mathcal{C}$ is proposed.⁵³ For the chain $c = (i, j, k)$, first j chooses to accept or reject clearing the chain, and then k chooses whether to accept or reject clearing the chain.⁵⁴ If either j or k chooses to reject, the allocation remains unchanged and the game proceeds to the next time period with the new state $(\mathbf{D}, \mathcal{C} \setminus \{c\})$. If both j and k choose to accept, then we *clear* the (i, j, k) chain obtaining a new allocation $\bar{\mathbf{D}} = (\bar{d}_j^i)_{i,j \in I}$, where

$$\bar{d}_n^m = \begin{cases} d_n^m - e(c) & \text{if } m = i \text{ and } n = j \\ d_n^m - e(c) & \text{if } m = j \text{ and } n = k \\ d_n^m + e(c) & \text{if } m = i, n = k \text{ and } i \neq k \\ d_n^m & \text{otherwise,} \end{cases}$$

and the game proceeds to the next time period with the new state $(\bar{\mathbf{D}}, \mathcal{H}(\bar{\mathbf{D}}))$.⁵⁵ If the state is of the form (\mathbf{D}, \emptyset) , then the game ends and final utilities are realized at the allocation \mathbf{D} .

Note that it is clear the game is finite: Clearing a chain always strictly decreases the total number of chains. Hence, for each new allocation formed during the *rescontre* process, either the process ends as no chains are cleared or continues to a new allocation with a strictly smaller number of chains; hence, the process must complete in a finite number of steps.

We abstract from mechanisms that consider both cycles and chains as clearing a chain can be modeled as the successive clearing of cycles. For instance, suppose that the set of agents is $I = \{i, j, k\}$, and that $d_j^i = 100$, $d_k^j = 100$, $d_i^k = 100$, with no other creditor-debtor

⁵³For technical simplicity, we assume that agents know which chain will be proposed for each state $(\mathbf{D}, \mathcal{C})$.

⁵⁴We assume that the approval of i is not required for clearing the chain (i, j, k) , since clearing that chain does not change the set of debts that are owed to i .

⁵⁵Also note that chains which were previously rejected may now be reconsidered, since the new state includes all of the feasible chains at $\bar{\mathbf{D}}$.

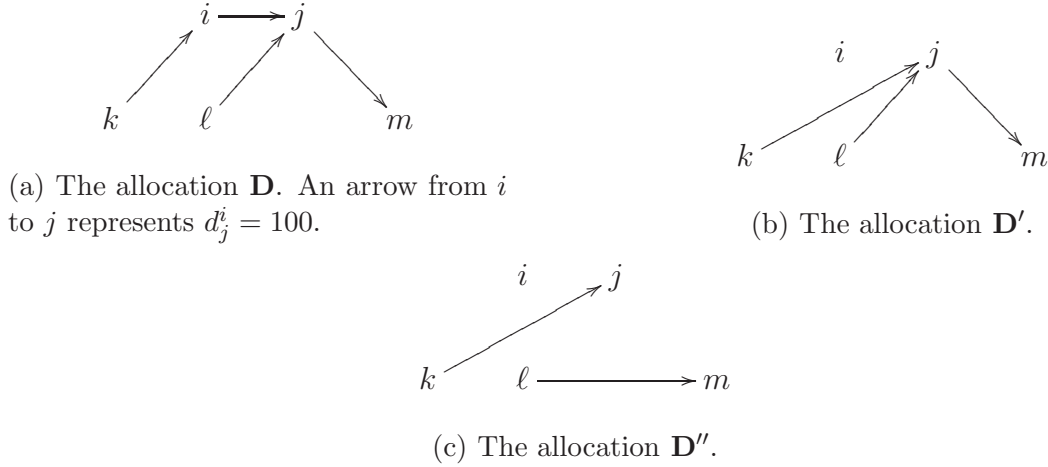


Figure 3: An example economy.

relationships. Then the cycle (i, j, k) can be cleared by first clearing the chain (i, j, k) and then clearing the newly formed chain (i, k, i) .

Example 2. Consider the example economy given in Figure 3a. Let $\alpha_i > \alpha_j > \alpha_k > \alpha_\ell > \alpha_m$. The initial set of chains is given by $\mathcal{H}(\mathbf{D}) = \{(k, i, j), (i, j, m), (\ell, j, m)\}$. Suppose that the chain (k, i, j) is chosen. It is clearly optimal for i to agree to clear this chain, since that leaves i with no outstanding debtors. Suppose that j also agrees to clear the chain. The new allocation \mathbf{D}' is depicted in Figure 3b, and the set of chains is given $\mathcal{H}(\mathbf{D}') = \{(k, j, m), (\ell, j, m)\}$. Suppose that the chain (ℓ, j, m) is chosen, and that both j and m agree to clear the chain. The new allocation \mathbf{D}'' is depicted in Figure 3c, but the set of chains is now empty, i.e., $\mathcal{H}(\mathbf{D}'') = \emptyset$; hence, the rescontre chains mechanism ends.

Note that the outcome in Example 2 is Pareto efficient; there is no way to rearrange the debts so that every agent is better off. Unfortunately, the rescontre chains mechanism does not necessarily produce a Pareto efficient allocation when positions are not balanced, as the following example demonstrates.

Example 3. Consider the allocation \mathbf{D} depicted in Figure 4a. Suppose that j is a better debtor than i (i.e., $\alpha_j < \alpha_i$), and that the quality of i as a debtor is such that $\alpha_\ell < 2\alpha_i <$



(a) The allocation \mathbf{D} . An arrow from i to j represents $d_i^j = 100$.

(b) A Pareto improvement over the allocation of Figure 4a.

Figure 4: An example economy for which the rescontre chains mechanism does not produce a Pareto efficient allocation.

$\alpha_\ell + \alpha_j$.⁵⁶ The set of chains for the allocation \mathbf{D} is $\mathcal{H}(\mathbf{D}) = \{(i, j, k)\}$. Hence, at the state $(\mathbf{D}, \mathcal{H}(\mathbf{D}))$, the chain (i, j, k) is proposed. If either j or k rejects clearing the chain, the game ends with the original allocation. If both accept clearing the chain, we obtain the new allocation $\bar{\mathbf{D}}$ induced by clearing (i, j, k) , and the game ends as the resulting allocation is such that $\mathcal{H}(\bar{\mathbf{D}}) = \emptyset$. However, k is worse off at \mathbf{D} than at $\bar{\mathbf{D}}$ as i is a worse debtor than j ; hence, in the unique subgame perfect Nash equilibrium, the final allocation is simply the original allocation \mathbf{D} . However, the allocation \mathbf{D} is not Pareto efficient. In particular, it is Pareto dominated by the allocation depicted in Figure 4b: j is better off since he now holds 100 in debt from ℓ instead of 200 in debt from i , and k is better off since i is a better debtor than ℓ .

Nevertheless, we will show that subgame perfect Nash equilibria of the rescontre mechanism do satisfy a notion of constrained Pareto efficiency. An allocation is *constrained Pareto efficient* if there does not exist a sequence of agents $i_1, i_2, \dots, i_Z, i_{Z+1} = j$ such that $d_{i_{z+1}}^{i_z} > 0$ for all $i = 1, \dots, Z$ and $\alpha_{i_1} \leq \alpha_{i_Z}$; in other words, an allocation \mathbf{D} is constrained Pareto efficient if there does exist a sequence of debtors and a final creditor j such that the first debtor is a better credit risk than the last debtor. If such a sequence existed, a Pareto superior allocation can be obtained by canceling the intervening debts and moving the debt of i_1 to j . Constrained Pareto efficiency is a weaker notion of efficiency as it does not consider the possibility to “trade” sets of debts. Figure 4a is an example of an allocation that is not Pareto efficient but is constrained Pareto efficient.

⁵⁶For instance, we could have $\alpha_j = \frac{2}{100}$, $\alpha_i = \frac{3}{100}$, and $\alpha_\ell = \frac{5}{100}$.

Constrained Pareto efficiency is a considerably weaker notion than Pareto efficiency; however, as shown in Theorem 2 below, obtaining Pareto efficiency in all cases requires the use of sophisticated financial transactions where at least two agents agree to new debt contracts *simultaneously*. That is, if \mathbf{D} is a constrained Pareto efficient outcome and $\bar{\mathbf{D}}$ is a Pareto improvement, then there exist at least two distinct pairs of agents who were not in a debtor–creditor relationship under \mathbf{D} but are in such a relationship under $\bar{\mathbf{D}}$. Thus, our definition of constrained Pareto efficiency is motivated by the historical fact that only the delegation of debts in one direction can be observed in the historical record. More complex transactions which would involve the trading of debts (e.g., the transaction described in Example 3) cannot be documented.⁵⁷

Theorem 2. *Suppose that \mathbf{D} is constrained Pareto efficient and that $\bar{\mathbf{D}}$ is a Pareto improvement over \mathbf{D} . Then there exist two distinct pairs of agents (i, j) and (m, n) such that $d_j^i < \bar{d}_j^i$ and $d_n^m < \bar{d}_n^m$.*

Example 3, depicted in Figure 4, provides an illustration of Theorem 2: the Pareto improvement requires the formation of new debt contracts between both the pair i and k and the pair ℓ and j . By contrast, clearing a cycle does not require forming any new debt relationships and clearing a chain requires forming at most one new debt relationship.

We now show that, in general, clearing chains is enough to obtain constrained Pareto efficient outcomes.

Theorem 3. *Suppose that each agent has a distinct default probability, i.e., $\alpha_i = \alpha_j$ if and only if $i = j$ for all $i, j \in I$. A subgame perfect pure strategy Nash equilibrium of the rescontre chains mechanism exists for any initial allocation, and, for any pure strategy Nash equilibrium, the final allocation is constrained Pareto efficient.*

⁵⁷The restriction to transactions that involve at most one new debt contract might be explained by the complexity of such procedures, but more likely can be understood as an institutional limitation under which agents were not allowed to trade debt relationships (Usher 1943; Van der Wee 1963; North 1981; Munro 1994). Allowing the transfer of even one debt in a clearing step during the recontre procedure was already a considerable weakening of the regulations against the tradability of debt.

Unfortunately, when agents may have identical default probabilities, not all subgame perfect Nash equilibria are necessarily constrained Pareto efficient: Consider an example where $I = \{i, j, k\}$ and an allocation such that the only chain is (i, j, k) . Further suppose that $\alpha_i = \alpha_j$. Then it is an equilibrium for k to reject clearing the chain (i, j, k) since he is indifferent between a debt from i and a debt from j .

Hence, we augment the utility of each agent by having each agent place a (very small) weight on the welfare of others. We define the *augmented utility* of agent i for an allocation \mathbf{D} as

$$\bar{u}^i(\mathbf{D}) = u^i(\mathbf{D}) + \epsilon \sum_{j \in I} u^j(\mathbf{D})$$

for some $\epsilon > 0$. This assumption is motivated by the fact that participating merchants had an interest in ensuring the smooth working of the mechanism and creating high clearing rates; thus, our assumption that agents accept clearing chains for which they are indifferent seems reasonable in this context. This assumption is equivalent to the assumption that agents accept clearing a chain when indifferent; such an assumption is standard in models of bargaining, where agents are assumed to accept proposals for which they are indifferent.

Theorem 4. *Suppose that agents' payoffs are given by their augmented utility functions for any $\epsilon > 0$. A subgame perfect pure strategy Nash equilibrium of the rescontre chains mechanism exists for any initial allocation, and, for any pure strategy Nash equilibrium, the final allocation is constrained Pareto efficient.*

Theorems 3 and 4 imply that the rescontre mechanism will be quite efficient in reducing the amount of outstanding debt (and credit) in the economy. Moreover, the voluntary nature of chain clearing also incentivizes agents to truthfully reveal their assets and liabilities.

The intuition behind why any pure strategy equilibrium of the game induced by the rescontre chains mechanism achieves a constrained Pareto efficient allocation differs from that of Theorem 1. Suppose there existed a pure strategy equilibrium such that the final allocation \mathbf{D} was not constrained Pareto efficient; consider the last proposed chain (i, j, k)

such that i is a (weakly) better debtor than j .⁵⁸ Given the resulting subgame after j or k rejects the proposed chain fixes the allocation at \mathbf{D} , both j and k are strictly better off by changing their strategy to approve the (i, j, k) chain and then not approving any other chain they are a part of. This results in an allocation that is strictly better for j and k than \mathbf{D} (no matter the strategies of other players); hence the original strategy profile did not constitute a subgame perfect Nash equilibrium. Note that this logic holds even if each agent is not fully informed regarding the allocation, but only aware of the chains in which he is involved (and whether the mechanism will end if that agent rejects clearing a chain).

Moreover, it is sufficient that agents play myopic, “locally-maximizing” strategies. Suppose that, for any proposed chain (i, j, k) , j always agrees to clear the chain (as it reduces the total liabilities owed to j), and k agrees to clear the chain if and only if i is a (weakly) better debtor than j . In this case, the final outcome of the mechanism must be constrained Pareto efficient.⁵⁹

One particular case of interest is the case where there are only two types of debtors. If some merchants were known to be reliable, while other merchants’ ability to repay was unknown, then there would effectively be two types of debtors. In this case, any constrained Pareto efficient outcome is Pareto efficient, leading to the following corollary.

Corollary 1. *Suppose that agents’ payoffs are given by their augmented utility functions for any $\epsilon > 0$; moreover, suppose that, for all $i \in I$, $\alpha_i = \alpha'$ or $\alpha_i = \alpha'' > \alpha'$. A subgame perfect pure strategy Nash equilibrium of the rescontre chains mechanism exists for any initial allocation, and, for any such equilibrium, the final allocation is Pareto efficient.*

The Pareto efficiency of final allocations in this setting follows from the fact there are only two types of debtors. In the final allocation, the only agents who can be both creditors and debtors are good types, i.e., α' types, since a bad α'' type who was both a creditor and a

⁵⁸If an allocation is not constrained Pareto efficient, then such a chain must always exist; see the proof given in Appendix A.2.

⁵⁹Note, however, that such a strategy may not be optimal for a fully rational k —a fully rational k may reject the chain (i, j, k) even if $\alpha_i < \alpha_j$, if there exists another chain (h, j, k) such that $\alpha_h < \alpha_i$ and k expects (h, j, k) to clear instead.

debtor would find any of his creditors willing to take on any of his debtors. Moreover, agents who are both creditors and debtors have only debts from bad α'' types, since debts from good α' types would have been happily cleared. Hence, the only remaining chains in the economy would be of the form (i, j, k) , where i is a bad type ($\alpha_i = \alpha''$) and j is a good type ($\alpha_j = \alpha'$). It then follows that the final outcome is Pareto efficient, since the only way to clear any more debts would be have an agent give up a good α' debtor for a bad α'' debtor.

We also note that the order in which clearing chains are proposed affects the final distribution of debits and credits; there are many possible constrained Pareto efficient allocations for a given initial allocation \mathbf{D} . With balanced positions, this was not the case: regardless of the order of proposals, in any Nash equilibrium, the final allocation is the same. By contrast, with imbalanced positions, order matters. For instance, if a merchant has both many creditors and debtors, his creditors likely have preferences over possible chains involving that merchant and the creditor, and so the order that chains are proposed will (partially) determine the final allocation.

4 Historical and Economic Context

We now consider our theoretical results in their economic and historical context. Theorems 1, 3 and 4 predict that outcomes will be constrained Pareto efficient and, if the initial allocation is balanced, the outcome will be fully efficient. Hence, we can rationalize the decentralized clearing procedures used, as their decentralized nature largely did not impede their efficiency. In order for the clearing process to be successful, it was sufficient that the merchants met at a specific time and cleared the chains and cycles: no centralized authority was necessary.

However, the evolving legal regulations also reflect the challenges faced over the centuries related to the use of the clearing mechanism. For instance, one type of regulation deals with the separation of questionable bills of exchange. It is crucial that no questionable bills are transferred since merchants may then unknowingly take on disputed debts. Hence, it is quite

natural that legislation governing debt clearing required that only acknowledged debts could be exchanged. A similar issue arose with regards to the use of endorsed bills when bills of exchange started to be transferable in the late 16th century (van der Wee 1977, pp. 149-63). However, many fairs, such as those in Frankfurt and Lyon, did not allow the use of endorsed bills during the *rescontre* procedure when endorsed bills were first introduced, and only later allowed the use of endorsed bills from places where the endorsement was legally accepted (Haristoy 1906, pp. 309f.; Schneider 1989, 1991; Brübach 1994, p. 171-4).

Fairs also differed in whether they required that merchants' positions were balanced. When positions are balanced, our Theorem 1 shows that a merchant should always be willing to participate in any clearing cycle. This greatly simplifies the clearing process, and may explain why the regulations at the Besancon and Lyon fairs required balanced positions, i.e., why these early fairs insisted on the payment of cash or the satisfactory drawing of new bills at the end of every fair.⁶⁰ Moreover, regulations requiring balanced positions increased the willingness of merchants to participate in a given clearing chain or cycle. An alternative regulation to achieve the goal of balanced positions was to ask each merchant for an *ex ante* deposit; this method was eventually used in Besancon.

The requirement of balanced positions is also related to contemporary work on payment mechanisms: Studies of modern payment mechanisms have also identified that the enforcement of debt positions at the end of a dynamic process is critical to enable the granting of credit earlier and for more heterogeneous debtors (Freeman 1996; Kiyotaki and Moore 2000; Mills 2004). Furthermore, these studies have argued that balancedness in net clearing mechanisms reduces the incentive to default (Kahn et al. 2003).

However, when positions are imbalanced and enforcement policies are limited, simply clearing cycles is no longer sufficient. When positions are imbalanced, a merchant may wish to use clearing cycles: by doing so, the merchant acquires no new debtors, so even a bankrupt clearing partner will not harm him, as the regulations clearly state that merchants are not

⁶⁰A merchant unable to comply with this regulation was declared bankrupt and excluded from any future clearing processes.

responsible for debts cleared at the fair. But, when positions are imbalanced, cycles are not sufficient to obtain even constrained Pareto efficient outcomes even in relatively simple economies, and so merchants may also use chains to improve the final allocation.

However, when a merchant clears debts through chains in addition to cycles, he must be careful not to pick a new partner who, as is written in the merchant booklet, turns out to be a bad debtor. This explains why rules in the exchange regulations typically gave each merchant the freedom to not participate in any particular chain.

We have seen that the *rescontre* mechanism will produce a Pareto efficient clearing if either positions are balanced (Theorem 1) or there is limited heterogeneity among debtors (Corollary 1). However, there are a few instances in the historical record in which the mechanism broke down, such as in the second half of the 16th century in Lyon. If there are both good and bad debtors, a simple increase in the number of bad debtors would not necessarily decrease the amount of mutual clearing. However, an increase in the heterogeneity of debtors could reduce the amount of mutual clearing, possibly leaving participants with long, unresolved clearing chains (due to the hierarchy of preferences over debtors). How much clearing efficiency will be reduced by such heterogeneity cannot be predicted *ex ante* since it depends on the complexity of the debt relationships; however, Example 3 illustrates that such heterogeneity can significantly reduce clearing efficiency. Thus, a shock which creates greater heterogeneity among debtors will likely reduce the efficiency of the clearing mechanism, lowering the incentives for merchants to participate in the mechanism.

Finally, an important property of the *rescontre* mechanism is the ability for participants to transact with both familiar clearing partners (whose creditworthiness they can judge) and anonymous partners with whom they can clear debts via cycles (so that no expertise is necessary). This property is important as cross-border enforcement was limited and trade was strongly based on reputation. Thus, the *rescontre* mechanism allowed a merchant to engage in transactions with others not personally known to him, facilitating impersonal market interactions. This is in line with the work of Greif (2002, 2006) who identified other

institutions which also facilitated more impersonal market interactions.⁶¹

5 A Comparative Perspective: Clearing Clubs and Clearing Banks

5.1 Clearing Clubs: Restricted Access and Self-Selection

Clearing clubs also used the *rescontre* mechanism for debt clearing. However, entry into such clubs was member-determined: hence, current members could ensure that any new member was of high quality.⁶² It has also been documented that some merchants attempted to establish clearing clubs at exchange fairs: The first efforts at creating clearing clubs that can be documented were by Italian bankers, who started small clearing fairs during the 1570s, when the Spanish bankruptcy led to uncertainty among merchant bankers. Additionally, in Frankfurt in 1642, twenty-three merchant bankers decided to meet regularly to clear their liabilities (Denzel 1994, pp. 316f). Similar agreements can be found among banking houses to clear liabilities during the late 18th century: For instance, in London in 1775, several private bankers agreed to form a clearing club, the London Clearinghouse, to settle their liabilities. Before this agreement, bankers had to present checks to each other and to settle individually. Anecdotal evidence suggests that cash messengers from different banks first met informally over breakfast and offset the liabilities so that they only had to cash in imbalanced positions later on. The clearing procedure used by the banking houses was in fact very similar to the

⁶¹In particular, Greif identifies for the period of investigation the community responsibility system as an institutional solution that enabled the transition from a reputation-based system to a more anonymous trade-based system with limited centralized outside enforcement. The institution of community responsibility allows for collective liability among members of a community, such as a city-state, where all members of that community are collectively liable for the actions of any given member. This allowed merchants to easily enter new markets, as their new business partners had access to an effective enforcement mechanism through the community responsibility institution.

⁶²Later clubs codified loss-sharing rules, as discussed by Moser (1998); however, such rules were not used by clearing clubs during the time period we study.

procedure known from the clearing fairs.⁶³

Since the membership of clearing clubs was determined by the members themselves, clearing clubs could exclude bad and uncertain debtors, reducing the heterogeneity of the debtor pool. If all members were of the same quality, Corollary 1 ensures that using the rescontre mechanism to clear debts among the members of a clearing club would lead to efficient final allocations. In fact, if all members of the clearing club were of the same quality, a stronger result can be shown: there exists a subgame perfect Nash equilibrium of the rescontre chains mechanism where every proposed chain is accepted; moreover, in any equilibrium, each agent is either only a creditor, only a debtor, or neither. This ensures not only that the final allocation is efficient, but that the final allocation minimizes the amount of money which must be transferred to clear the remaining debts.

Corollary 2. *Suppose that agents' payoffs are given by their augmented utility functions for any $\epsilon > 0$; moreover, suppose that, for all $i \in I$, $\alpha_i = \hat{\alpha}$. A subgame perfect pure strategy Nash equilibrium of the rescontre chains mechanism exists for any initial allocation, and, for any such equilibrium, the final allocation is Pareto efficient.*⁶⁴

Corollary 2 implies that clearing clubs had a strong incentive to exclude potential members who were lower quality debtors than current members, as admitting them could disrupt the smooth functioning of the clearing process. Scholars who study modern payment mechanisms have also emphasized the role of restricted access. Moreover, they have stressed the role of alternative institutions such as banks. These alternative solutions can provide clearing services to less trustworthy customers as they have lower monitoring costs (Kahn and Roberds 2002).

⁶³For details of the London Clearinghouse, see Seyd (1872, pp. 52ff). Such clearing clubs were also formed in Dublin in 1845, in New York in 1853, in Paris and Vienna in 1872, and in Berlin in 1884 (Koch, 1983).

⁶⁴Note that is a special case of Corollary 1.

5.2 The Introduction of Central Banks

Many prominent trading centers introduced central banks in order to facilitate the clearing process (among other reasons). Central banks acted as third parties which handled clearing for their customers. The establishment of such banking institutions can be documented back to the late Middle Ages. We find both private banks in the form of deposit banks and public banks founded by the political authorities of merchant cities mainly in Spain and Italy (Usher 1943; Braudel 1966; Lane 1937; Luzzato 1934; Lane and Mueller 1985). However, neither type of bank was successful as a clearing institution. Many deposit banks went bankrupt during the 15th and 16th centuries due to risky business activities. This reduced the incentives of merchants to hold accounts and clear their liabilities via the bank. Public banks were mainly established to manage (and often rehabilitate) its public finances, i.e., to handle tax income streams, issue bonds, and pay interest. Hence, many central banks took on the role of clearing liabilities in order to increase liquidity or control financial transactions, and not to support credit transactions related to trade. Consequently, merchants' trust in these public banks was limited, and was further reduced by unexpected policy decisions regarding issuance of new debts and the restriction or even cessation of payment. Hence, local authorities often had to force local merchants to clear via these banks.⁶⁵

The 17th century ushered in a new era of public banks. During this period, there were still many attempts to establish public banks to rehabilitate state finances with the help of private capital contributions—and many of them failed, including banks in Leipzig, Vienna, and Berlin (Denzel 2012). However, a number of the new public banks were created to support credit transactions and create monetary stability. Serving as a clearing house was one of the central purposes of these banks: The Rialto Bank of Venice, founded by the end of the 16th century, has been identified as the starting point of this movement (Luzzato 1934; Lane 1937; Lane and Mueller 1985). The bank of Amsterdam, founded during the 17th century, was also

⁶⁵In-depth studied examples include the public banks in the kingdom of Aragonia—in particular in Barcelona—and the bank of San Giorgio in Genoa: see Usher (1943), Felloni (2006), and Sieveking (1934a).

successful and is the most studied bank of this type (Van Dillen 1934; van der Wee 1977; de Vries and van der Woude 1997; Gillard 2004; Quinn and Roberds 2006, 2007; Dehing 2012). Other examples include the public exchange banks of Hamburg and Nuremberg (Sieveking 1934b; Fuchs 1954; Denzel 2012). However, important clearing locations such as Lyon and Frankfurt did not establish such banks. In Genoa, the introduction of a successful public clearing bank took until the end of the 17th century (Gianelli 2006).

Many of these banks were started on the initiative of merchants; however, their success as clearing institutions could only be established over time. While these banks were often granted a monopoly over the clearing process, this monopoly had to be enforced again and again, and in many cases weakened over time. This can be documented for all four banks of this early movement.⁶⁶ Thus, even during this period, merchants continued to use the *rescontre* procedure in these locations. Most of these banks were originally designed as pure exchange banks; however, they also regularly engaged in some kind of profitable banking activity. Of the early four exchange banks, only Nuremberg acted as a pure exchange bank (Denzel 2012). By contrast, the bank of Hamburg offered credit to private customers and the town of Hamburg very soon after its founding (Sieveking 1934b). The bank of Amsterdam lent money to selected customers during its early years as well: to the town of Amsterdam, to the Dutch East Indian Company, and to the Amsterdam Lending bank. In addition, the bank of Amsterdam made high profits by buying and selling gold and silver (van Dillen 1934; Dehing 2012). Finally, little is known about the early years of the Rialto bank. However, its successor—the Giro Bank, which was started in 1619—was involved in lending activities as well (Luzzato 1934).

A number of questions arise from our description of the historical evolution of these institutions: What factors explain the success or failure of the central banks which developed during this period? In particular, how did the heterogeneity of debtors affect the success of these banks? Why were merchants unwilling to use many of these central banks, instead

⁶⁶For Venice, see Lattes (1869, pp. 170f.). For Amsterdam, see van Dillen (1934, p. 92) and Sneller (1940, pp. 157-160). For Hamburg and Nuremberg, see Prausnitz (1928, p. 26) and Neuhaus (1892, p. 12).

preferring the rescontre procedure? Conversely, why did some of these banks, such as the Amsterdam Exchange Bank, succeed?

We now extend our model of debt clearing to consider clearing bank mechanisms in order to answer these questions.

5.3 The Central Bank Mechanism

We model the *central bank mechanism* as a game where each agent i simultaneously chooses a subset of his debtors $J_i \subseteq \{j \in I : d_i^j > 0\}$; the debts from agents in J_i to i are *submitted* to the central bank. The central bank then assumes the responsibility for paying i the amount $\sum_{j \in J_i} d_i^j$.

Denote the set of agents who go bankrupt (and hence do not pay off their debts) by B . We model the bank as collecting from every debtor who does not default, and then paying each of its creditors (i.e., agents who submitted debts to the bank) from that pool; each creditor is paid in proportion to the sum of the debts collected.⁶⁷ Hence, the utility of agent i is given by

$$-\varphi \frac{\sum_{k \in I} \sum_{j \in J_k} \mathbb{1}_{j \in B} d_k^j}{\sum_{k \in I} \sum_{j \in J_k} d_k^j} \sum_{j \in J_i} d_i^j - \sum_{j \in I \setminus J_i} \mathbb{1}_{j \in B} d_i^j$$

where $\varphi \geq 0$ characterizes the loss to a creditor from the bank either not paying him or only paying him after a large delay. If $\varphi \approx 1$, then the creditor loses in proportion to the amount lost by the bank, while if $\varphi \approx 0$, the creditor is almost fully and quickly repaid in all cases. Note that the payoff to i from debts i submits to the bank depends on debt contracts that do not involve agent i ; the central bank effectively collectivizes the debt holdings that are turned over to it.

⁶⁷Public banks typically did not formulate procedures for managing default in advance. However, from analyzing the case of the bankruptcy of the Nuremberg bank in 1635, we know that the town of Nuremberg paid back the debts over two years, in weekly payments to each customer (Denzel 2012). The Hamburg bank closed temporarily in 1672 due to expected liquidity problems: In that case, the customers did not get access to their money for a period of one year. Only a few exceptions of small monetary transfers can be documented (Pohl 1986). Hence, we model the loss to merchants from this delayed payment as a pro rata loss which depends only on the total amount collected by the bank.

Hence, recalling that the probability of default by agent j is given by α_j , the expected utility of agent i is given by

$$-\varphi \frac{\sum_{k \in I} \sum_{j \in J_k} \alpha_j d_k^j}{\sum_{k \in I} \sum_{j \in J_k} d_k^j} \sum_{j \in J_i} d_i^j - \sum_{j \in I \setminus J_i} \alpha_j d_i^j$$

This expression provides the mapping from strategies to payoffs, thus completing the description of the game.

Our first result shows that, in such a setting, the only debts turned over to the central bank are of the lowest quality.

Theorem 5. *For any $\varphi \geq 1$, in any Nash equilibrium of the game induced by the central bank mechanism, the expected utility of each agent $i \in I$ is*

$$\sum_{j \in I} \alpha_j d_i^j,$$

i.e., the expected utility he would receive by keeping his initial set of debts.

Consider an agent i with a debtor ℓ who is the highest type, i.e., $\alpha_\ell \geq \alpha_k$ for all $k \in I$, and suppose that losses to creditors are large if the bank defaults, i.e., $\varphi = 1$. If i submits a set of debtors to the central bank which includes agent ℓ , his expected loss from the debt owed by ℓ is

$$\frac{\sum_{k \in I} \sum_{j \in J_k} \alpha_j d_k^j}{\sum_{k \in I} \sum_{j \in J_k} d_k^j} d_i^\ell.$$

By contrast, if agent i does not submit the debt from agent ℓ , his expected loss from the debt owed by agent ℓ is simply $\alpha_\ell d_i^\ell$. But since $\alpha_\ell \leq \alpha_k$ for all $k \in I$, it must be that i obtains a lower expected loss from not submitting the debt from ℓ ; essentially, since ℓ is the most trustworthy agent, he is more trustworthy than any weighted average of other agents' trustworthiness, and so i is better off not submitting his debt from ℓ to the central bank. But then, since no agent will submit debts from ℓ , using the same logic we can determine that the expected return from submitting debts of the second-most trustworthy agent ℓ' must also

be lower than the expected return from not submitting the debt from ℓ' . Iterating this logic yields the result that only debts from the lowest type will be submitted to the central bank. Hence, since only debts from the type most likely to default are submitted to the bank, no agent will do better than if he simply did not disclose any debts to the bank.

Intuitively, the use of the bank “unravels” since the highest quality debts are not submitted to the bank but rather held privately; hence, the second highest quality debts are not submitted either, and so on, until only the lowest quality debts are reported to the bank. Thus a bank which works simply as an exchange mechanism cannot succeed if clearing participants are heterogeneous and losses are distributed proportionally. For instance, the bank of Nuremberg, which was designed as a pure exchange bank, had to cope with these issues. After a promising start in 1611, the bank’s activity soon declined: The clearing volume and deposits steadily shrunk, and contemporary witnesses report that participants only brought their worst debts to clear. In 1635, the bank declared bankruptcy with hardly any transaction volume or deposits left. Later efforts to reestablish the bank during the 17th century were similarly unsuccessful (Fuchs 1954, pp. 55-8; Denzel 2012).

However, a central bank may have some systematic advantages over the *rescontre* mechanism. For instance, it may be the case that the bank can generate revenue from its holdings; a debtor may deposit money to pay off debts while the creditor to that debtor may only withdraw his money from the bank later, and the bank may be able to make a profit by holding the money in the interim. As discussed in Section 5.2, such business practices can indeed be documented for many of the central banks, including the banks of Amsterdam and Hamburg. Profits earned by the bank in this way may be used to help pay off debts the bank accepted that ultimately default.

We now consider a central bank for whom clearing debts is intrinsically profitable. We denote by r the bank’s *rate of return* on debts it collects. Hence, the total revenue available

to a central bank to compensate creditors is given by

$$(1+r) \sum_{k \in I} \sum_{j \in J_k} \mathbb{1}_{j \notin B} d_k^j;$$

recall that B is the (random) set of agents who go bankrupt. In this case, the payoff to agent i from submitting the set of debts J_i to the central bank depends on which agents go bankrupt: If few agents go bankrupt, then the bank will be able to fully pay agent i . On the other hand, if many agents go bankrupt, the ability of the bank to pay agent i will be limited. Hence, the expected utility of agent i is given by

$$-\mathbb{E} \left[\max \left\{ 0, \varphi \left(1 - \frac{(1+r) \sum_{k \in I} \sum_{j \in J_k} \mathbb{1}_{j \notin B} d_k^j}{\sum_{k \in I} \sum_{j \in J_k} d_k^j} \right) \right\} \right] \sum_{j \in J_i} d_i^j - \sum_{j \in I \setminus J_i} \alpha_j d_i^j.$$

This expression reflects the fact that, if the bank makes money from its operations, then the bank keeps any money not necessary to pay those who cleared their debts with the bank.

This additional revenue generation implies that the conclusion of Theorem 5 no longer holds. In particular, a positive rate of return may incentivize some higher quality debtholders to bring their debts to the bank. This, in turn, may induce yet higher quality debtholders to bring their debts as well. Hence, for high enough rates of return, the bank may become the favored mechanism for debt clearing.

Theorem 6. *There exists a rate of return \bar{r} such that, for all $r \geq \bar{r}$, for all $\varphi \in [0, 1]$, it is a Nash equilibrium of the game induced by the central bank mechanism for each agent to submit all of his debts, i.e., $J_i = \{j \in I : d_i^j > 0\}$ for all $i \in I$.*

However, even for such sufficiently high rates of return, there can exist multiple equilibria. This implies that the ability of a central bank to coordinate behavior may prove important: such coordination may allow the bank to achieve an outcome where many high-quality debts are brought to the bank. Example 4 below provides an illustration.

Example 4. Consider a setting with $I = \{i, j, k, \hat{i}, \hat{j}, \hat{k}\}$ where $d_i^i = d_j^j = d_k^k = 100$ with no

other debts, and suppose that $\alpha_i = \alpha_j = .92$, while $\alpha_k = .9$. Further suppose that $r = .1$ and $\varphi = 1$. Then there exist multiple equilibria in the game induced by the central banking mechanism.

In one equilibrium, only agent k submits his (poorer quality) debt to the central bank. Since k holds the worst-quality debt, it is clearly optimal (and, in fact, a weakly dominant strategy) for him to bring his debt to the bank. Now consider the problem facing agent i . If he does not submit the debt from \hat{i} to the bank, his expected payoff is $100 \cdot .92 = 92$, while his expected payoff from submitting his debt to the bank is

$$.9 \cdot .92(100) + .1 \cdot .92 \left(\frac{1}{2}(100)(1+r) \right) + .9 \cdot .08 \left(\frac{1}{2}(100)(1+r) \right) + .1 \cdot .08(0) = 91.82; \quad (2)$$

the first term is the state of the world where neither agent defaults, the second is the state of the world where only \hat{k} defaults, the third is the state of the world where \hat{i} defaults, and the last is the state of the world where both agents default. Since $91.82 < 92$, it is optimal for i to not submit his debt from \hat{i} to the central bank. An analogous argument holds for j .

However, there exists another equilibrium where all three of i , j , and k submit their debts to the bank. It is clearly optimal for k to do so, but i and j must consider the expected return from clearing their debt through the bank against holding the debt privately. A calculation similar to (2) yields the result that the expected return to i from submitting the debt from \hat{i} to the bank is ≈ 92.3 ; this expected return is higher since the bank now holds the debt from \hat{j} . Since this is greater than 92, it is indeed a best response for both i and j to submit their debts to the bank, given that the other is doing so as well.⁶⁸

The above example shows that multiple equilibria are possible; moreover, under some equilibria, the bank is much more successful than others. The key issue for the bank is whether it can attract high quality debts. If so, then other creditors will be happy to bring their debts to the bank, since high quality debts will provide working capital to the bank

⁶⁸In fact, multiple equilibria exist for all $r \in (.083, .122)$.

and allow the bank to make up for possible defaults more easily. Therefore, a bank, by clearing high quality debts, becomes a more attractive institution, which in turn brings in more high quality debts for clearing. Hence, if a central bank was able to profit from the clearing process, and it could ensure high rates of payment by attracting enough high quality transactions, agents would naturally want to use it.

We can, in fact, document such a process taking place at the Amsterdam bank. The Bank of Amsterdam was already profitable by the first half of the 17th century. Indeed, we can observe a high participation rate by the financial elite and merchant capitalists of the time (Dehing 2012). These merchants not only cleared their positions, but also kept deposits in the bank, and the number of account holders grew rapidly during the 17th century. These historical observations support the theoretical predictions of a successfully functioning central clearing bank.

However, the question of why high quality creditors were initially willing to clear their debts through the exchange bank remains open. One explanation is that the equilibrium in which high quality debts were brought to the bank was made focal by the credible trust in the business activities of the bank and the exercise of enforcement powers by the city of Amsterdam. Amsterdam guaranteed the deposits, overdrawing was penalized,⁶⁹ and both Amsterdam and the Dutch state had economic enforcement power which clearly extended beyond their local borders. Moreover, after an improvement in accounting methods reduced the number of overdrafts (Dehing 2012, Chapter 3), from 1683 onwards a clearing credit could be obtained cheaply by selling coins at a fixed price and repurchasing them from the bank within six months' time for $\frac{1}{8}$ to $\frac{1}{4}$ percent of the value of the coins (van Dillen 1934, pp. 102f.). Allowing those who cleared through the bank credit at advantageous terms can be thought of in the model as increasing r , the rate of return on debts deposited at the bank. The town of Amsterdam also forced merchants to clear credit transactions over a certain minimum level, which may have led to a critical mass of participants (van Dillen

⁶⁹Overdrawing was penalized after it had been tolerated due to primitive book keeping techniques during the first half of the 17th century (Willemsen 2009, p. 89)

1934). Finally, wealthy merchants were very likely induced to participate by the fact that they were closely linked to the local government, which was substantially influenced by the merchant elite of the time. A similar story (albeit on a smaller scale) can also explain the success of the Hamburg bank: in particular, the Hamburg bank also provided clearing credit through the purchasing and reselling of silver bullion.

By contrast, the bank of Nuremberg was not successful: Although the bank started with a reasonable number of customers, including the wealthy local capitalists of Nuremberg who helped to establish the bank, the bank did not succeed. The bank was not able to profit from its deposits and did not offer any clearing credit to its depositors; moreover, the town representatives also took money out of the bank without informing the customers or paying any interest (Fuchs 1954, pp. 55-8; Denzel 2012). Finally, economic conditions in Nuremberg were significantly worse than in the booming towns of Amsterdam and Hamburg: this led to a higher likelihood of default by merchants, exacerbating the inability of the bank to pay off its depositors.

A number of prominent cities did not start a bank at all: Velde (2009) argues that, in the case of Lyon, the lack of trust in such a public institution could have been a major reason why no such clearing institution was established in the first place. He argues that merchants would had to reveal all their (sensitive) business activities to the bank, and merchants were simply unwilling to do so.⁷⁰

In sum, a central bank does have some advantages over the *rescontre* procedure: such a bank can always produce a Pareto efficient outcome when agents turn over their entire debt portfolio to the bank. Such a bank also effectively reallocates debts among all the merchants, creating many new debt contracts simultaneously; the intuition for why this is necessary is similar to that of Theorem 2, which implies that creating at least two new debt contracts simultaneously is necessary to obtain Pareto efficiency in general. However, such a bank also needs to create trust: This enables the bank to attract “good” customers to

⁷⁰During the early 18th century, Marberger (1717, pp. 98f.) comments on the problem of trusting public exchange banks (Schneider 1991, p. 142).

create liquidity and make profits to insure against losses. By contrast, the *rescontre* process does not require trust in a central institution,⁷¹ but may not be able to achieve full Pareto efficiency, particularly when debtors are very heterogeneous. However, the *rescontre* process can still function in the presence of such heterogeneity, and so it was used for clearing debts between merchants and merchant bankers from all over Europe. By contrast, public exchange banks focused on the clearing of debts at permanent market places in merchant cities and often restricted the access to residents (e.g., in Hamburg and Nuremberg). Such restrictions were easier to implement in the early Modern period since economic activities started to concentrate in big cities such as Amsterdam, London, or Hamburg and foreign merchants became permanent residents. This increased trust in these foreign merchants (and thus in the bank that held their debts).⁷² The economic concentration also enabled exchange banks to grow and to exploit economies of scale, making them more attractive relative to the *rescontre* process.⁷³ For instance, whereas the Italian clearing fairs during the late 16th and early 17th centuries counted around 50-120 merchant bankers as core participants, the number of account holders at the Bank of Amsterdam increased from a few hundred at the beginning of the 17th century to more than twenty-five hundred at the end of the century (Dehing 2012, p. 141).⁷⁴

6 Conclusion

This paper examines the evolution of *rescontre*, a mechanism for clearing non-tradable debts during the late medieval and early modern period in Europe. We constructed a simple theoretical model of the mechanism and showed that the mechanism could lead to very high

⁷¹Of course, the lack of such a trusted central institution also reduced the access of participants to any form of liquidity in the first place.

⁷²In terms of our model, this is equivalent to lowering the default probability of the worst debtors.

⁷³In terms of our model, this is equivalent to increasing r at the central bank.

⁷⁴However, comparing these numbers is difficult as, for the Bank of Amsterdam, a large majority of account holders were private customers and only a smaller, but nevertheless critical share—about 5 percentage points by the end of the 17th century (Dehing 2012, p. 138)—were financial intermediaries, while at the clearing fairs the majority were merchant bankers who likely represented several other customers.

clearing rates without strong outside enforcement needs, helping us to understand why this mechanism was so commonly used throughout Europe for debt clearing.

We first examined the history of the mechanism. We analyzed the early emergence of this mechanism at Late Medieval markets and fairs and showed how the clearing algorithm developed into a standardized procedure embedded in a legal framework during the early Modern period. Moreover, we demonstrated the use of this clearing procedure throughout Europe. Finally, we identified the main elements of the procedure, based on legal sources and writings of contemporary witnesses.

We then constructed a model of the mechanism and identified the equilibrium outcomes and related strategic behavior of participants. We showed that the functioning of the mechanism depends on the balance of positions of merchants *ex ante* and the use of clearing cycles or clearing chains. If positions are balanced, then efficient clearing is always achievable with clearing cycles. If positions are imbalanced, it is necessary to use clearing chains as well. Clearing a chain involves debt transfers which must be approved by all parties affected. We show that preferences over debtors influence the clearing behavior and type of efficiency which can be achieved. When there are only good and bad debtors, the clearing chains will lead to a Pareto efficient outcome in equilibrium. However, if more types are introduced, long chains may not be resolved due to the differences in debtor quality, and only constrained Pareto efficient outcomes can be guaranteed.

We then discussed these theoretical properties in their economic and historical context. We showed that the *rescontre* procedure was effective in the settings in which it was used; moreover, a number of regulations, such as requirements for participation, facilitated the proper functioning of the procedure. Finally, we compared the *rescontre* procedure to alternative clearing options found in the historical literature: clearing clubs and public exchange banks. Clearing clubs created restricted access to the *rescontre* procedure and reduced the heterogeneity among participants. This reduced the set of participants but likely increased the efficiency of the clearing outcome. By contrast, exchange banks could always efficiently

clear the reported debts. However, they had to create additional institutional mechanisms to convince merchants to bring their liabilities to the bank. Participants had not only to trust the banking institution, but also faced the risk of loss by pooling their liabilities with other participants. We showed that if sufficiently high quality liabilities were brought to clear, the bank could generate a sufficiently high profit to insure against the clearing risk and the public exchange bank could succeed in the long run. Otherwise, potential participants would be unwilling to use the bank and would prefer the *rescontre* procedure.

Our results also open up new questions for future research. While this paper focused on the institutional mechanism used for clearing, future work may wish to provide a complementary empirical analysis of the trading and clearing behavior of merchants in the context of different institutional solutions. Finally, it is well known that all these clearing locations were linked and embedded in a common network: the evolution and robustness of this network should be further investigated.

7 References

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A Proofs

A.1 Proof of Theorem 1

We start by proving the first claim of the theorem. Consider the strategy profile where each agent accepts every proposed cycle; each agent achieves his highest possible payoff under this strategy profile, and so no deviation strategy (in any subgame) can lead to a higher payoff; hence, this strategy profile is a subgame perfect Nash equilibrium.

We now prove the second claim of the theorem. Suppose, by way of contradiction, there exists a subgame perfect Nash equilibrium such that the final allocation is not $(0)_{i,j \in I}$ with positive probability. Consider any path that obtains a final state (\mathbf{D}, \emptyset) ; note that the set of still viable cycles must be empty at a final state. Consider the state $(\bar{\mathbf{D}}, \mathcal{C})$ one step before. There are two cases:

1. If $\mathbf{D} \neq \bar{\mathbf{D}}$, then a cycle was cleared in that last step, implying the final set of viable cycles is $\mathcal{Y}(\mathbf{D})$, i.e., $\mathcal{Y}(\mathbf{D}) = \mathcal{C}$. But, since positions are balanced (by assumption) and $\mathbf{D} \neq (0)_{i,j \in I}$, the set $\mathcal{Y}(\mathbf{D})$ is nonempty, a contradiction.⁷⁵
2. If $\mathbf{D} = \bar{\mathbf{D}}$, then no cycle was cleared in the last step, and the state in the penultimate step was of the form $(\bar{\mathbf{D}}, \{c\})$. Hence, the cycle $c = (i_1, \dots, i_Z)$ was proposed in the last step and the clearing of that cycle was rejected by at least one agent. Consider

⁷⁵To see that $\mathcal{Y}(\mathbf{D})$ is nonempty, note that there exists agents i_1 and i_2 such that $d_{i_1}^{i_2} > 0$. But since positions are balanced, there must exist an agent i_3 such that $d_{i_2}^{i_3} > 0$. More generally, since positions are balanced, if $d_{i_{K-1}}^{i_K} > 0$ for agents i_{K-1} and i_K , then there must exist an agent i_{K+1} such that $d_{i_K}^{i_{K+1}} > 0$. Hence, we can construct a sequence of agents $i_1, i_2, \dots, i_K, i_{K+1}$ where $K = |I|$. Hence, there must exist x, y such that $1 \leq x < y \leq K + 1$ such that $i_x = i_y$. But then (i_x, \dots, i_y) is a cycle.

the strategy of agent i_K conditional on agents i_1, \dots, i_{Z-1} approving the clearing of the cycle. Suppose agent i_Z rejects clearing the cycle; agent i_Z would be strictly better off by approving the cycle and rejecting any future cycles of which he is a part. Since i_Z rejects any future cycles, the only effect on his utility through this deviation is the direct effect of clearing this cycle. Moreover, approving the cycle increases the utility of i_Z as $\alpha_{Z-1} > 0$. Hence, in any subgame perfect equilibrium, agent i_Z will not reject clearing this cycle (conditional on other agents accepting). Analogous logic shows that agent i_z , for $z = 1, \dots, Z - 1$, will not reject clearing this cycle conditional on agents i_1, \dots, i_{z-1} approving clearing the cycle. Hence, it can not be part of a subgame perfect equilibrium for any agent to reject clearing the cycle c at the state $(\bar{\mathbf{D}}, \{c\})$.

A.2 Proof of Theorems 3 and 4

Existence of a subgame perfect pure strategy Nash equilibrium of finite perfect information games is well-known: see, e.g., Theorem 3.3 of Fudenberg and Tirole (1991). Hence, we prove that any subgame perfect pure strategy Nash equilibrium obtains a constrained Pareto efficient outcome.

We first show that if an allocation \mathbf{D} is not constrained Pareto efficient, then there must exist at least one chain $(i, j, k) \in \mathcal{H}(\mathbf{D})$ such that $\alpha_i \leq \alpha_j$, i.e., i is a (weakly) better debtor than j . Suppose \mathbf{D} is not constrained Pareto efficient; then, by definition, there exists a sequence of agents i_1, \dots, i_Z, i_{Z+1} such that $\alpha_{i_1} \leq \alpha_{i_Z}$. Now suppose that no chain $(i, j, k) \in \mathcal{H}(\mathbf{D})$ exists such that $\alpha_i \leq \alpha_j$. If $\alpha_{i_1} \leq \alpha_{i_2}$, then the chain (i_1, i_2, i_3) satisfies our requirement; thus, $\alpha_{i_1} > \alpha_{i_2}$. More generally, then, if $\alpha_{i_{z-1}} \leq \alpha_{i_z}$, then (i_{z-1}, i_z, i_{z+1}) is a chain that satisfies our requirement; thus, $\alpha_{i_{z-1}} > \alpha_{i_z}$. Hence, by transitivity, if no such chain exists we have that $\alpha_{i_1} > \alpha_{i_Z}$, contradicting the assumption that $\alpha_{i_1} \leq \alpha_{i_Z}$.

Now, suppose the game ends at an outcome that is not constrained Pareto efficient. After the last chain is cleared along the equilibrium path, the state is given by the final allocation $\bar{\mathbf{D}}$ and the set of chains generated by that allocation $\mathcal{H}(\bar{\mathbf{D}})$. Given the argument above, there

must exist at least one chain $(i, j, k) \in \mathcal{H}(\bar{\mathbf{D}})$ such that $\alpha_i \leq \alpha_j$. Consider the last rejected chain (i, j, k) such that $\alpha_i \leq \alpha_j$, i.e., that i is a weakly better debtor than j . Consider the choice of k conditional on the chain (i, j, k) being accepted by i and j . Approving the chain and rejecting any future chains weakly increases the utility of k and strictly increases the utility of j , thus strictly increasing the (augmented) utility of k ; ⁷⁶ hence, k rejecting the chain at this point can not be an equilibrium strategy. Now consider the strategy of j conditional on i accepting the chain; j knows in equilibrium that if he approves the chain, k will approve the chain as well. Approving the chain and rejecting any future chains makes j strictly increases the utility of j ; hence, j rejecting the chain at this point can not be an equilibrium strategy. Hence, both j and k will accept the chain, a contradiction.

A.3 Proof of Corollary 1

We show that, in the two-type setting, any constrained Pareto efficient allocation is Pareto efficient; the result then follows from Theorem 4.

Consider any constrained Pareto efficient allocation \mathbf{D} and suppose, by way of contradiction, there exists a Pareto improvement $\bar{\mathbf{D}}$. We first make three observations:

1. Consider agent i such that i is only a debtor, i.e., $d_i^j = 0$ for all $j \in I$. Then, since $\bar{\mathbf{D}}$ must leave the net position of i unchanged, i is not strictly better off under $\bar{\mathbf{D}}$.
2. Consider agent i such that i is only a creditor, i.e., $d_j^i = 0$ for all $j \in I$. Then, since $\bar{\mathbf{D}}$ must leave the net position of i unchanged, i must hold at least as much high quality debt under $\bar{\mathbf{D}}$ as under \mathbf{D} for $\bar{\mathbf{D}}$ to be a Pareto improvement.
3. Consider agent i such that i is both a creditor and a debtor. First, note that $\alpha_i = \alpha'$; if instead $\alpha_i = \alpha''$, then there would exist a chain (h, i, j) where $\alpha_h \leq \alpha_i = \alpha''$, and so \mathbf{D} would not be constrained Pareto efficient. Moreover, if $d_i^h > 0$, then $\alpha_h = \alpha''$; otherwise, for some j such that $d_j^i > 0$, the chain (h, i, j) would imply that the allocation \mathbf{D} is

⁷⁶If $\alpha_i \neq \alpha_j$, then the utility of k is strictly increased by clearing this chain, since by assumption $\alpha_i \leq \alpha_j$.

not Pareto efficient. So i must also hold as much low-quality debt under $\bar{\mathbf{D}}$ as under \mathbf{D} .

It follows from the third observation that the quantity of lower quality debt is the same under \mathbf{D} as under $\bar{\mathbf{D}}$. Moreover, from the second observation, the amount of high quality debt must be the same, as all such debt is held by agents who are strictly creditors. Hence, the sum of the agents' utilities is the same under $\bar{\mathbf{D}}$ and \mathbf{D} —recall from the first observation that agents who are purely debtors have 0 utility under both allocations. But this contradicts the assumption that each agent is weakly better off and one agent is strictly better off under $\bar{\mathbf{D}}$ than under \mathbf{D} .

A.4 Proof of Theorem 5

Let $\varphi = 1$. Suppose some other such Nash equilibrium existed. For the expected payoff of an agent i to be different than if that agent did not submit any debts, i must have submitted at least one debt to the bank, and there must exist at least one other agent j who submitted at least one debt of a different quality than one debt submitted by i . That is, there must exist $i, j, \hat{i}, \hat{j} \in I$ such that $\hat{i} \in J_i$, $\hat{j} \in J_j$, and $\alpha_{\hat{i}} \neq \alpha_{\hat{j}}$. Let $\hat{i} = \arg \min_{\hat{k} \in \{k \in I : k \in J_j \text{ for some } j \in I\}} \alpha_{\hat{k}}$, and let i be some agent such that $\hat{i} \in J_i$; in other words, let \hat{i} be the best debtor for whom some creditor i submits the debt from \hat{i} .

The expected utility of i under this equilibrium is given by

$$u^i(\{J_k\}_{k \in I}) = -\varphi \frac{\sum_{k \in I} \sum_{j \in J_k} \alpha_j d_k^j}{\sum_{k \in I} \sum_{j \in J_k} d_k^j} \sum_{j \in J_i} d_i^j - \sum_{j \in I \setminus J_i} \alpha_j d_i^j.$$

If i submits $\hat{J}_i \equiv J_i \setminus \{\hat{i}\}$ instead, i has an expected utility of

$$u^i(\{J_k\}_{k \in I}) = -\varphi \frac{\sum_{k \in I} \sum_{j \in J_k} \alpha_j d_k^j - \alpha_{\hat{i}} d_{\hat{i}}^{\hat{i}}}{\sum_{k \in I} \sum_{j \in J_k} d_k^j - d_{\hat{i}}^{\hat{i}}} \sum_{j \in J_i \setminus \{\hat{i}\}} d_i^j - \sum_{j \in I \setminus (J_i \setminus \{\hat{i}\})} \alpha_j d_i^j.$$

Thus the change in utility of i from choosing to not submit the debt from \hat{i} is, after some

algebraic manipulation, given by

$$\frac{d_i^{\hat{i}} \left(\sum_{k \in I \setminus \{i\}} \sum_{j \in J_k} d_k^j \right) \left(\sum_{k \in I} \sum_{j \in J_k} \alpha_j d_k^j - \alpha_i d_i^{\hat{i}} - \alpha_i \left(\sum_{k \in I} \sum_{j \in J_k} d_k^j - d_i^{\hat{i}} \right) \right)}{\left(\sum_{k \in I} \sum_{j \in J_k} d_k^j \right) \left(\sum_{k \in I} \sum_{j \in J_k} d_k^j - d_i^{\hat{i}} \right)}. \quad (3)$$

Since $\alpha_i \leq \alpha_k$ for all $k \in \cup_{j \in I} J_j$, with at least one inequality holding strictly,⁷⁷ the expression

$$\left(\sum_{k \in I} \sum_{j \in J_k} \alpha_j d_k^j - \alpha_i d_i^{\hat{i}} - \alpha_i \left(\sum_{k \in I} \sum_{j \in J_k} d_k^j - d_i^{\hat{i}} \right) \right)$$

is positive. All the other multiplicative terms in (3) are also clearly positive.⁷⁸ Hence, i is better off not reporting the debt from \hat{i} .

A.5 Proof of Theorem 6

Suppose that r is large enough that if any debt submitted to the bank is collected, then the bank can pay off every creditor, i.e.,

$$(1 + r)d_i^h \geq \sum_{j \in I} \sum_{k \in I} d_k^j$$

for all $i, h \in I$ such that $d_i^h > 0$.⁷⁹ Consider any debt d_i^h ; in the state of the world in which h does not default, i would receive d_i^h from h if he does not submit the debt, but i will also receive d_i^h from the bank since it is sufficient for that one debt to clear for the bank to pay all of its creditors. In the state of the world in which h does default, i would not receive any transfer from h , and so submitting the debt to the bank would result in a (weakly) higher utility. Note that this argument does not depend on φ .

⁷⁷Recall that there must be at least one debt submitted of strictly lower quality.

⁷⁸Recall that at least one other agent is submitting a debt to the mechanism.

⁷⁹Note that this condition is sufficient but *not* necessary.

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C Proofs of Additional Results: Online Appendix

C.1 Proof of Theorem 2

For an allocation \mathbf{D} , let \mathbf{D}_j be the vector of debts owed to j , i.e., $\mathbf{D}_j = (d_j^i)_{i \in I}$, and let \mathbf{D}^j be the vector of debts owed by j , i.e., $\mathbf{D}^j = (d_i^j)_{i \in I}$.

Let $\succ_{\mathbf{D}}$ be a strict ordering over agents in I such that $i \succ_{\mathbf{D}} j$ if either

1. $\mathbf{D}^i = 0$, i.e., i is not a debtor, and $\mathbf{D}^j \neq 0$, i.e., j is a debtor, or
2. $\mathbf{D}^i, \mathbf{D}^j \neq 0$, i.e., both i and j are debtors, and $\alpha_i < \alpha_j$, i.e., i is a better debtor than j .

In other words, $\succ_{\mathbf{D}}$ is a strict ordering such that all non-debtors are ranked higher than all debtors, and, among any two debtors, a higher-quality debtor is ranked higher than a lower-quality debtor. Note that if $i \succ_{\mathbf{D}} j$, then $d_j^i = 0$ (i.e., i does not owe j) as \mathbf{D} is constrained Pareto efficient.⁸⁰

Since $\bar{\mathbf{D}}$ is a strict Pareto improvement over \mathbf{D} , the set $K \equiv \{k \in I : \mathbf{D}_k \neq \bar{\mathbf{D}}_k\}$ is non-empty; the set K is the set of agents who have a different vector of credits under $\bar{\mathbf{D}}$ than under \mathbf{D} . Let k be the highest-ranked element in K according to the ordering $\succ_{\mathbf{D}}$. As \mathbf{D} is constrained Pareto efficient, if $d_\ell^k > 0$ for some agent ℓ , then $\ell \succ_{\mathbf{D}} k$; but then $\mathbf{D}_\ell = \bar{\mathbf{D}}_\ell$ by the definition of k .

Hence, since \mathbf{D} and $\bar{\mathbf{D}}$ are net position equivalent, $\mathbf{D}_k \neq \bar{\mathbf{D}}_k$, and $d_\ell^k = \bar{d}_\ell^k$ for all ℓ such that $d_\ell^k > 0$, there must exist an agent i such that $\bar{d}_k^i > m_k^i$. Hence, (i, k) is the first set of

⁸⁰To see this fact, suppose not. Then $d_j^i > 0$ and we have that there exists an agent k such that $d_k^j > 0$ as i is a debtor and $i \succ_{\mathbf{D}} j$. But then \mathbf{D} is not constrained Pareto efficient, as clearing the chain (i, j, k) leads to a Pareto improvement (since $\alpha_i \leq \alpha_j$, as i and j are both debtors and $i \succ_{\mathbf{D}} j$).

agents that we have identified such that the former owes the latter more under the allocation $\bar{\mathbf{D}}$ than under \mathbf{D} .

Now, since \mathbf{D} and $\bar{\mathbf{D}}$ are net position equivalent for i , either

1. there exists an agent \hat{i} such that $\bar{d}_i^{\hat{i}} > d_i^{\hat{i}}$, in which case we are done as (\hat{i}, i) is the second set of agents that we have identified such that the former owes the latter more under the allocation $\bar{\mathbf{D}}$ than under \mathbf{D} , or
2. there exists an agent j_1 such that $\bar{d}_{j_1}^i < d_{j_1}^i$, i.e., an agent j_1 that i owes strictly less under the new allocation.

In the latter case, since \mathbf{D} and $\bar{\mathbf{D}}$ are net position equivalent for j_1 , either

1. there exists an agent \hat{i} such that $\bar{d}_{j_1}^{\hat{i}} > d_{j_1}^{\hat{i}}$, in which case we are done as (\hat{i}, j_1) is the second set of agents that we have identified such that the former owes the latter more under the allocation $\bar{\mathbf{D}}$ than under \mathbf{D} , or
2. there exists an agent j_2 such that $\bar{d}_{j_2}^{j_1} < d_{j_2}^{j_1}$, i.e., an agent j_2 that j_1 owes strictly less under the new allocation.

Again, in the latter case, since \mathbf{D} and $\bar{\mathbf{D}}$ are net position equivalent for j_2 , either

1. there exists an agent \hat{i} such that $\bar{d}_{j_2}^{\hat{i}} > d_{j_2}^{\hat{i}}$, in which case we are done as (\hat{i}, j_2) is the second set of agents that we have identified such that the former owes the latter more under the allocation $\bar{\mathbf{D}}$ than under \mathbf{D} , or
2. there exists an agent j_3 such that $\bar{d}_{j_3}^{j_2} < d_{j_3}^{j_2}$, i.e., an agent j_3 that j_2 owes strictly less under the new allocation.

Iterating this logic, either the theorem is true or there exists an infinite sequence j_1, j_2, j_3, \dots such that $\bar{d}_{j_{f+1}}^{j_f} < d_{j_{f+1}}^{j_f}$ for all $f = 1, 2, 3, \dots$. But then, since $\bar{d}_{j_{f+1}}^{j_f} \geq 0$ for all $f = 1, 2, 3, \dots$, we have that $d_{j_{f+1}}^{j_f} > 0$ for all $f = 1, 2, 3, \dots$. But then, since there are only a finite number of agents, there exists a cycle in \mathbf{D} , contradicting the constrained efficiency of \mathbf{D} .

C.2 Utility Functions: Online Appendix

We consider the case in which an agent's utility is linear in net profits; that is, conditional on not defaulting, the utility of agent i is the expected transfer from agents for whom i is a creditor minus any debts owed by i to other agents. We assume that the utility of i if he defaults is some arbitrary constant K .⁸¹ Then the *ex ante utility* of agent i for an allocation \mathbf{D} is given by

$$v^i(\mathbf{D}) \equiv (1 - \alpha_i) \left(\sum_{j \in I \setminus \{i\}} (1 - \alpha_j) d_i^j - \sum_{j \in I \setminus \{i\}} d_j^i \right) + \alpha_i K$$

We show that, when comparing allocations for which the net position of i does not change, the *ex ante utility* of i is an affine transformation of the expression for utility given in (1).

Lemma 1. *Suppose that, under the allocations \mathbf{D} and $\bar{\mathbf{D}}$, the net position of agent i is the same. Then*

$$v^i(\mathbf{D}) - v^i(\bar{\mathbf{D}}) = (1 - \alpha_i)(u^i(\mathbf{D}) - u^i(\bar{\mathbf{D}})).$$

⁸¹Note that default is exogenous and not chosen by the agent.

This lemma follows from a simple computation:

$$\begin{aligned}
 v^i(\mathbf{D}) - v^i(\bar{\mathbf{D}}) &= (1 - \alpha_i) \left(\sum_{j \in I \setminus \{i\}} (1 - \alpha_j) d_i^j - \sum_{j \in I \setminus \{i\}} d_j^i \right) - (1 - \alpha_i) \left(\sum_{j \in I \setminus \{i\}} (1 - \alpha_j) \bar{d}_i^j - \sum_{j \in I \setminus \{i\}} \bar{d}_j^i \right) \\
 &= (1 - \alpha_i) \left(\left(\sum_{j \in I \setminus \{i\}} (1 - \alpha_j) d_i^j - \sum_{j \in I \setminus \{i\}} d_j^i \right) - \left(\sum_{j \in I \setminus \{i\}} (1 - \alpha_j) \bar{d}_i^j - \sum_{j \in I \setminus \{i\}} \bar{d}_j^i \right) \right) \\
 &= (1 - \alpha_i) \left(\left(- \sum_{j \in I \setminus \{i\}} \alpha_j d_i^j + \sum_{j \in I \setminus \{i\}} d_j^j - \sum_{j \in I \setminus \{i\}} d_j^i \right) - \left(- \sum_{j \in I \setminus \{i\}} \alpha_j \bar{d}_i^j + \sum_{j \in I \setminus \{i\}} \bar{d}_j^j - \sum_{j \in I \setminus \{i\}} \bar{d}_j^i \right) \right) \\
 &= (1 - \alpha_i) \left(\left(- \sum_{j \in I \setminus \{i\}} \alpha_j d_i^j \right) - \left(- \sum_{j \in I \setminus \{i\}} \alpha_j \bar{d}_i^j \right) \right) \\
 &= (1 - \alpha_i) (u^i(\mathbf{D}) - u^i(\bar{\mathbf{D}})),
 \end{aligned}$$

where the fourth equality follows from the fact the net position of agent i is the same under \mathbf{D} and $\bar{\mathbf{D}}$.

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C Proofs of Additional Results: Online Appendix

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agents that we have identified such that the former owes the latter more under the allocation $\bar{\mathbf{D}}$ than under \mathbf{D} .

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1. there exists an agent \hat{i} such that $\bar{d}_{j_1}^{\hat{i}} > d_{j_1}^{\hat{i}}$, in which case we are done as (\hat{i}, j_1) is the second set of agents that we have identified such that the former owes the latter more under the allocation $\bar{\mathbf{D}}$ than under \mathbf{D} , or
2. there exists an agent j_2 such that $\bar{d}_{j_2}^{j_1} < d_{j_2}^{j_1}$, i.e., an agent j_2 that j_1 owes strictly less under the new allocation.

Again, in the latter case, since \mathbf{D} and $\bar{\mathbf{D}}$ are net position equivalent for j_2 , either

1. there exists an agent \hat{i} such that $\bar{d}_{j_2}^{\hat{i}} > d_{j_2}^{\hat{i}}$, in which case we are done as (\hat{i}, j_2) is the second set of agents that we have identified such that the former owes the latter more under the allocation $\bar{\mathbf{D}}$ than under \mathbf{D} , or
2. there exists an agent j_3 such that $\bar{d}_{j_3}^{j_2} < d_{j_3}^{j_2}$, i.e., an agent j_3 that j_2 owes strictly less under the new allocation.

Iterating this logic, either the theorem is true or there exists an infinite sequence j_1, j_2, j_3, \dots such that $\bar{d}_{j_{f+1}}^{j_f} < m_{j_{f+1}}^{j_f}$ for all $f = 1, 2, 3, \dots$. But then, since $\bar{d}_{j_{f+1}}^{j_f} \geq 0$ for all $f = 1, 2, 3, \dots$, we have that $d_{j_{f+1}}^{j_f} > 0$ for all $f = 1, 2, 3, \dots$. But then, since there are only a finite number of agents, there exists a cycle in \mathbf{D} , contradicting the constrained efficiency of \mathbf{D} .

C.2 Utility Functions: Online Appendix

We consider the case in which an agent's utility is linear in net profits; that is, conditional on not defaulting, the utility of agent i is the expected transfer from agents for whom i is a creditor minus any debts owed by i to other agents. We assume that the utility of i if he defaults is some arbitrary constant K .⁸¹ Then the *ex ante utility* of agent i for an allocation \mathbf{D} is given by

$$v^i(\mathbf{D}) \equiv (1 - \alpha_i) \left(\sum_{j \in I \setminus \{i\}} (1 - \alpha_j) d_i^j - \sum_{j \in I \setminus \{i\}} d_j^i \right) + \alpha_i K$$

We show that, when comparing allocations for which the net position of i does not change, the *ex ante utility* of i is an affine transformation of the expression for utility given in (1).

Lemma 1. *Suppose that, under the allocations \mathbf{D} and $\bar{\mathbf{D}}$, the net position of agent i is the same. Then*

$$v^i(\mathbf{D}) - v^i(\bar{\mathbf{D}}) = (1 - \alpha_i)(u^i(\mathbf{D}) - u^i(\bar{\mathbf{D}})).$$

⁸¹Note that default is exogenous and not chosen by the agent.

This lemma follows from a simple computation:

$$\begin{aligned}
 v^i(\mathbf{D}) - v^i(\bar{\mathbf{D}}) &= (1 - \alpha_i) \left(\sum_{j \in I \setminus \{i\}} (1 - \alpha_j) d_i^j - \sum_{j \in I \setminus \{i\}} d_j^i \right) - (1 - \alpha_i) \left(\sum_{j \in I \setminus \{i\}} (1 - \alpha_j) \bar{d}_i^j - \sum_{j \in I \setminus \{i\}} \bar{d}_j^i \right) \\
 &= (1 - \alpha_i) \left(\left(\sum_{j \in I \setminus \{i\}} (1 - \alpha_j) d_i^j - \sum_{j \in I \setminus \{i\}} d_j^i \right) - \left(\sum_{j \in I \setminus \{i\}} (1 - \alpha_j) \bar{d}_i^j - \sum_{j \in I \setminus \{i\}} \bar{d}_j^i \right) \right) \\
 &= (1 - \alpha_i) \left(\left(- \sum_{j \in I \setminus \{i\}} \alpha_j d_i^j + \sum_{j \in I \setminus \{i\}} d_j^i - \sum_{j \in I \setminus \{i\}} d_j^i \right) - \left(- \sum_{j \in I \setminus \{i\}} \alpha_j \bar{d}_i^j + \sum_{j \in I \setminus \{i\}} \bar{d}_j^i - \sum_{j \in I \setminus \{i\}} \bar{d}_j^i \right) \right) \\
 &= (1 - \alpha_i) \left(\left(- \sum_{j \in I \setminus \{i\}} \alpha_j d_i^j \right) - \left(- \sum_{j \in I \setminus \{i\}} \alpha_j \bar{d}_i^j \right) \right) \\
 &= (1 - \alpha_i) (u^i(\mathbf{D}) - u^i(\bar{\mathbf{D}})),
 \end{aligned}$$

where the fourth equality follows from the fact the net position of agent i is the same under \mathbf{D} and $\bar{\mathbf{D}}$.